

Interior Structure Modelling Review

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Motivation

- Interior parameters (composition, temperature structure) not directly observable
- Can observe parameters that reflect the interior (radius, T_{int} , Love number)
- Want to find parameters that match the observations
- Infer the past history of the planet's properties (for e.g. tides, mass loss)

Basic Equations

$$\frac{dP}{dm} = \frac{-Gm}{4\pi r^4}$$

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}$$

$$\rho = \text{EOS}(P, s, Z)$$

Thorngren

$$\frac{dP}{dr} = -\rho g,$$

$$\frac{dg}{dr} = -4\pi G\rho - \frac{2Gm}{r^3},$$

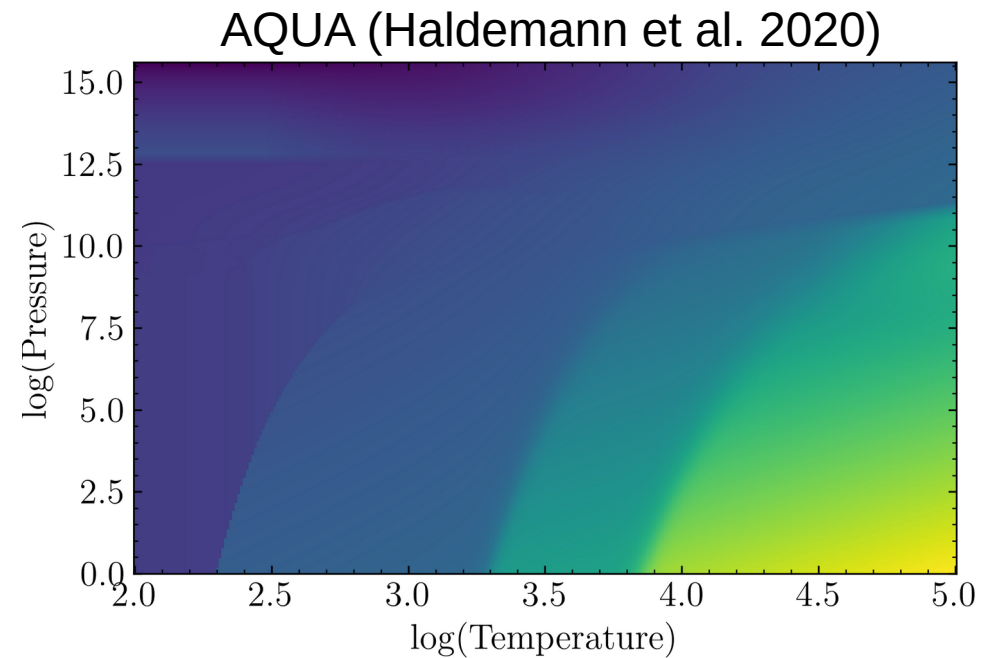
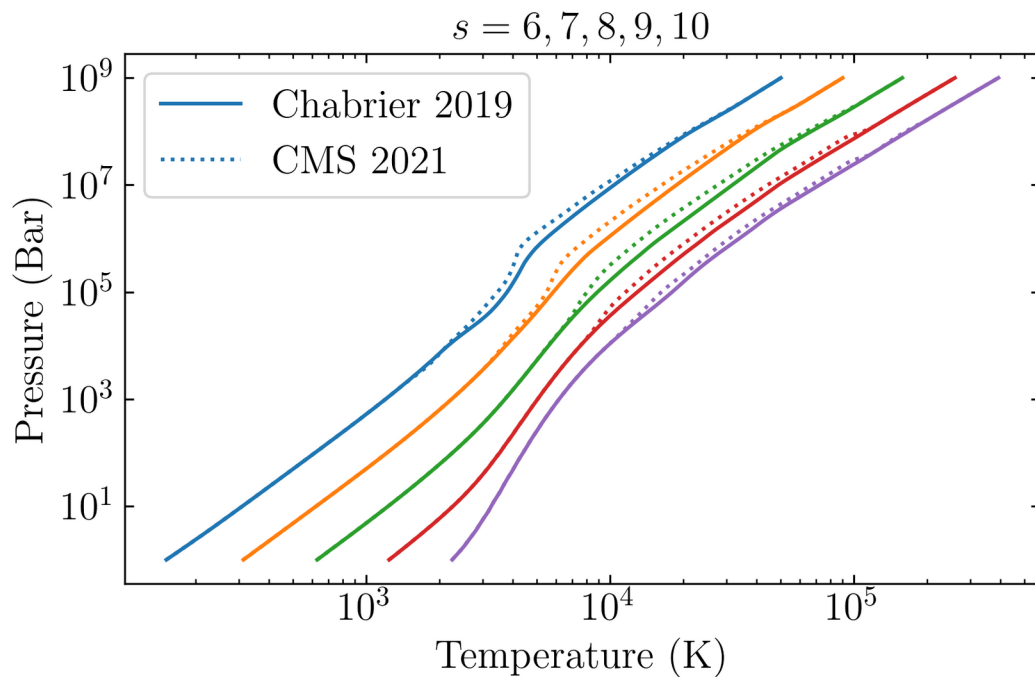
$$\frac{dT}{dr} = -g \frac{\gamma T}{\phi},$$

$$\left\{ \begin{array}{l} \phi = \frac{dP}{d\rho} \\ \gamma = V \left(\frac{dP}{dE} \right)_V \end{array} \right. ,$$

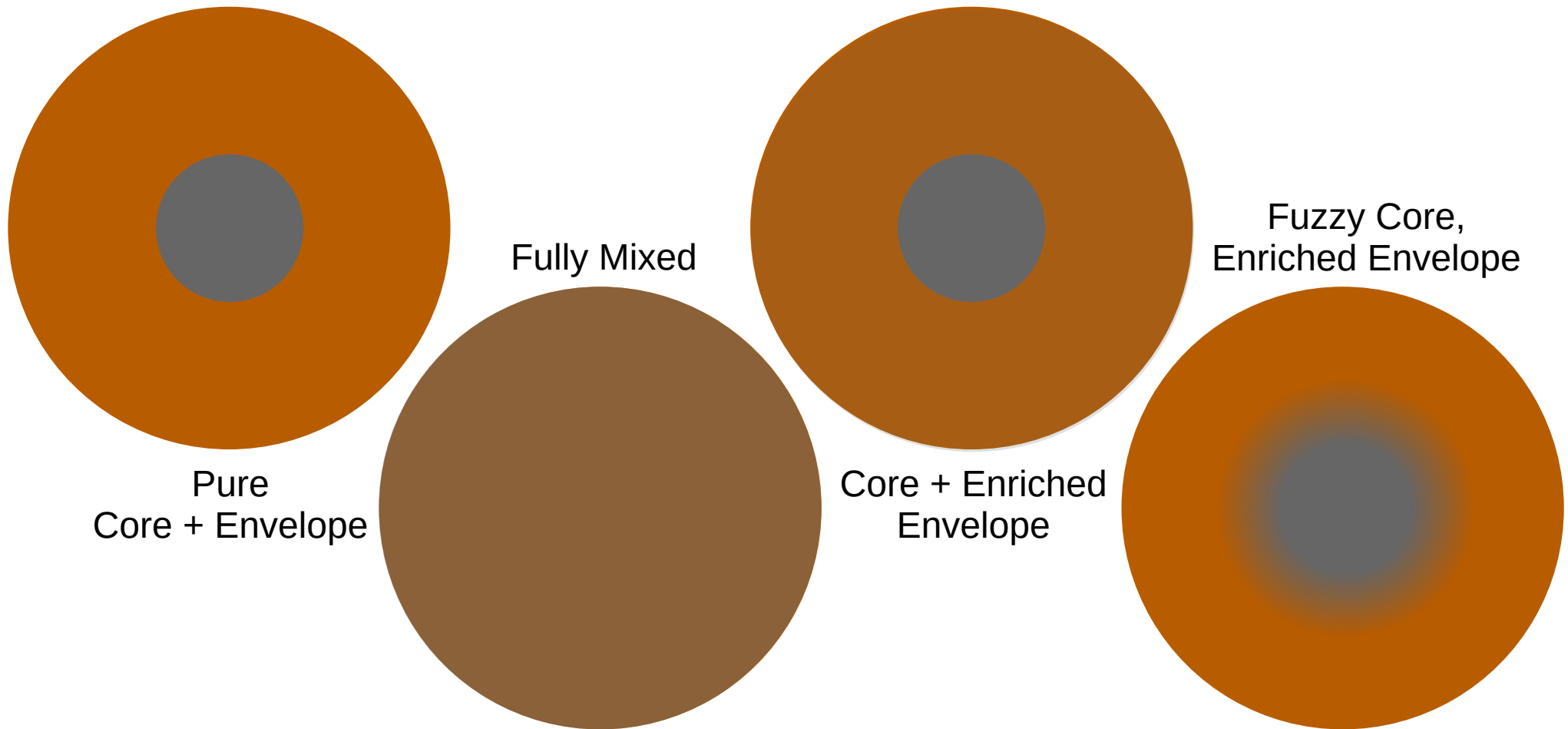
$$\frac{dm}{dr} = 4\pi r^2 \rho.$$

Acuña (2024)

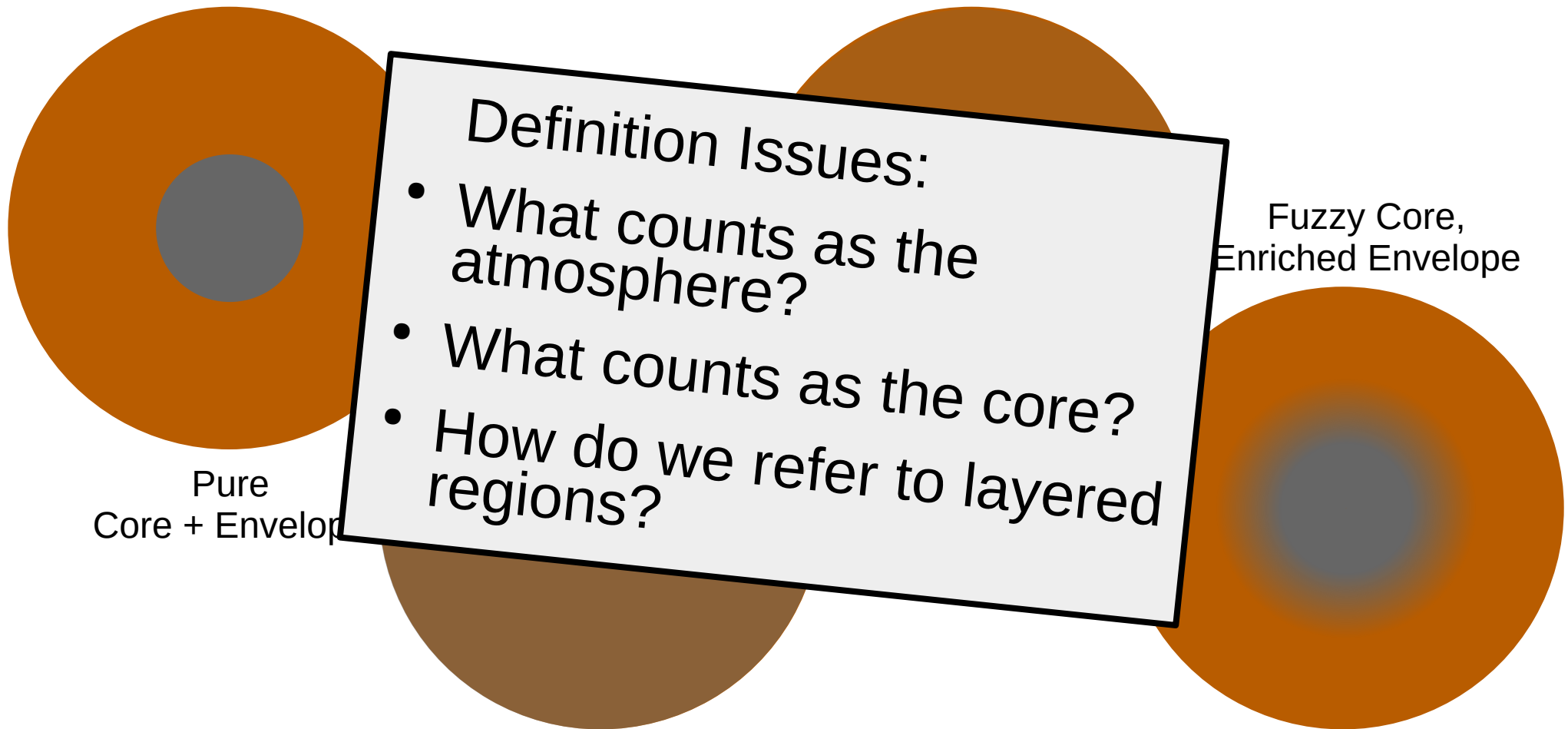
Equations of State



Metal Structuring



Metal Structuring



Temperature Structure

- Temperature structure reflects heat transfer
 - Conduction, convection, radiation

- Ledoux Criterion:

$$\nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\varphi}{\delta} \nabla_{\mu} \quad \delta := - \left(\frac{\partial \ln \rho}{\partial \ln T} \right), \quad \varphi := \left(\frac{\partial \ln \rho}{\partial \ln \mu} \right)$$

- Convection usually wins in the interior, unless composition gradients interfere
 - Rotation can also be a headache

Adiabatic Gradient

$$dU = TdS - PdV$$

$$du = Tds + \frac{P}{\rho^2}d\rho$$

$$dQ = TdS = 0 \qquad du = \frac{P}{\rho^2}d\rho$$

Adiabatic Gradient

$$dU = TdS - PdV$$

$$\left. \frac{\partial T}{\partial P} \right|_s = - \frac{\left. \frac{\partial s}{\partial P} \right|_T}{\left. \frac{\partial s}{\partial T} \right|_P} = - \frac{\frac{1}{\rho^2} \left. \frac{\partial \rho}{\partial T} \right|_P}{\frac{1}{T} \left(\left. \frac{\partial u}{\partial T} \right|_P - \frac{P}{\rho^2} \left. \frac{\partial \rho}{\partial T} \right|_P \right)}$$

$$= \frac{T \left. \frac{\partial \rho}{\partial T} \right|_P}{P \left. \frac{\partial \rho}{\partial T} \right|_P - \rho^2 \left. \frac{\partial u}{\partial T} \right|_P}$$

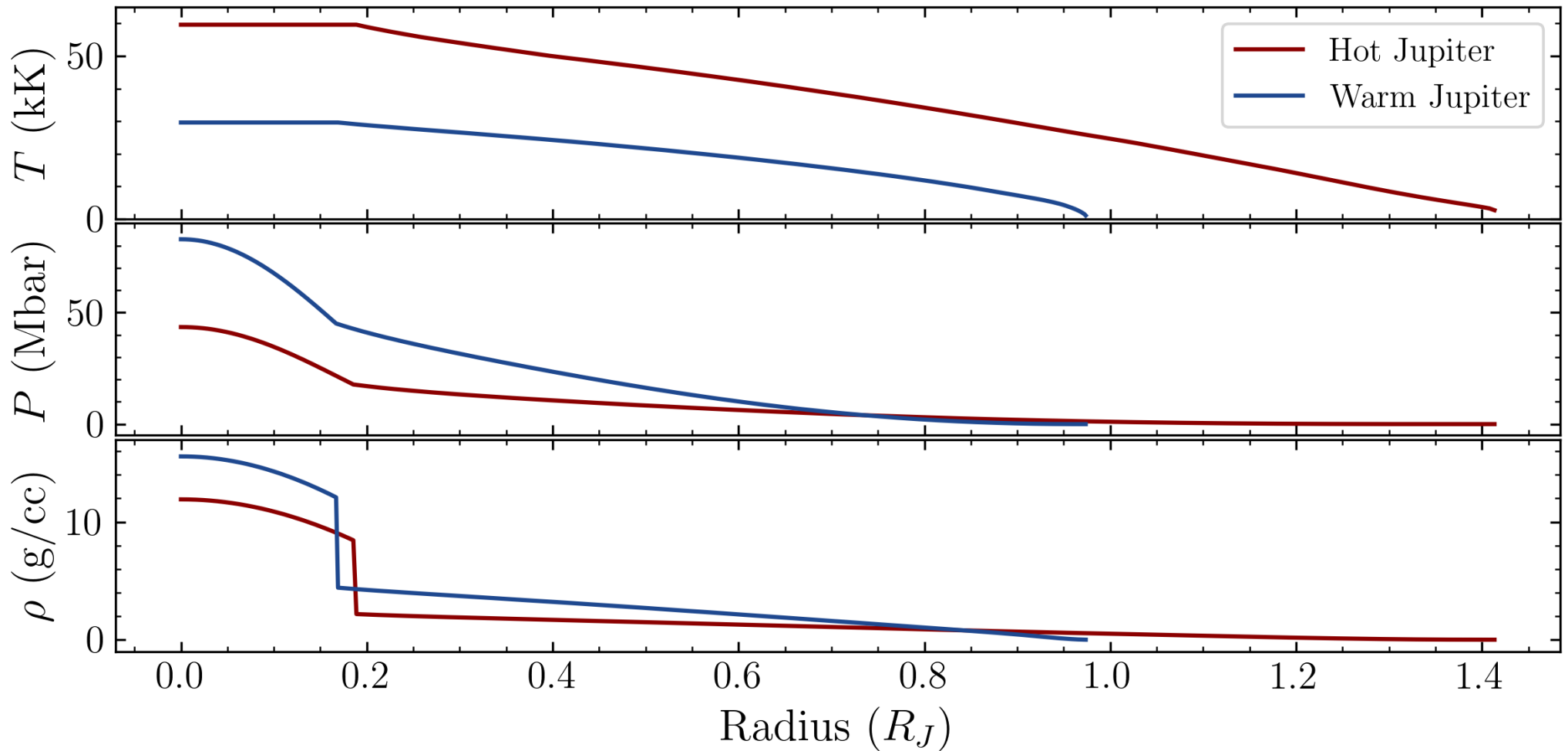
$dQ = T dS = 0$

$$du = \frac{1}{\rho^2} d\rho$$

Solving the Equations

- Local Adjustments
 - Explicitly track $T(r)$, transfer heat and update
 - Largely unavoidable if $Z(m)$ evolves
- Shooting to a fitting point:
 - Guess bounds, integrate, check, repeat
 - Used in stellar models
- Relaxation Solutions
 - Iteratively solve for $P(m)$, $\text{Rho}(m)$, $R(m)$...

Structure Solutions

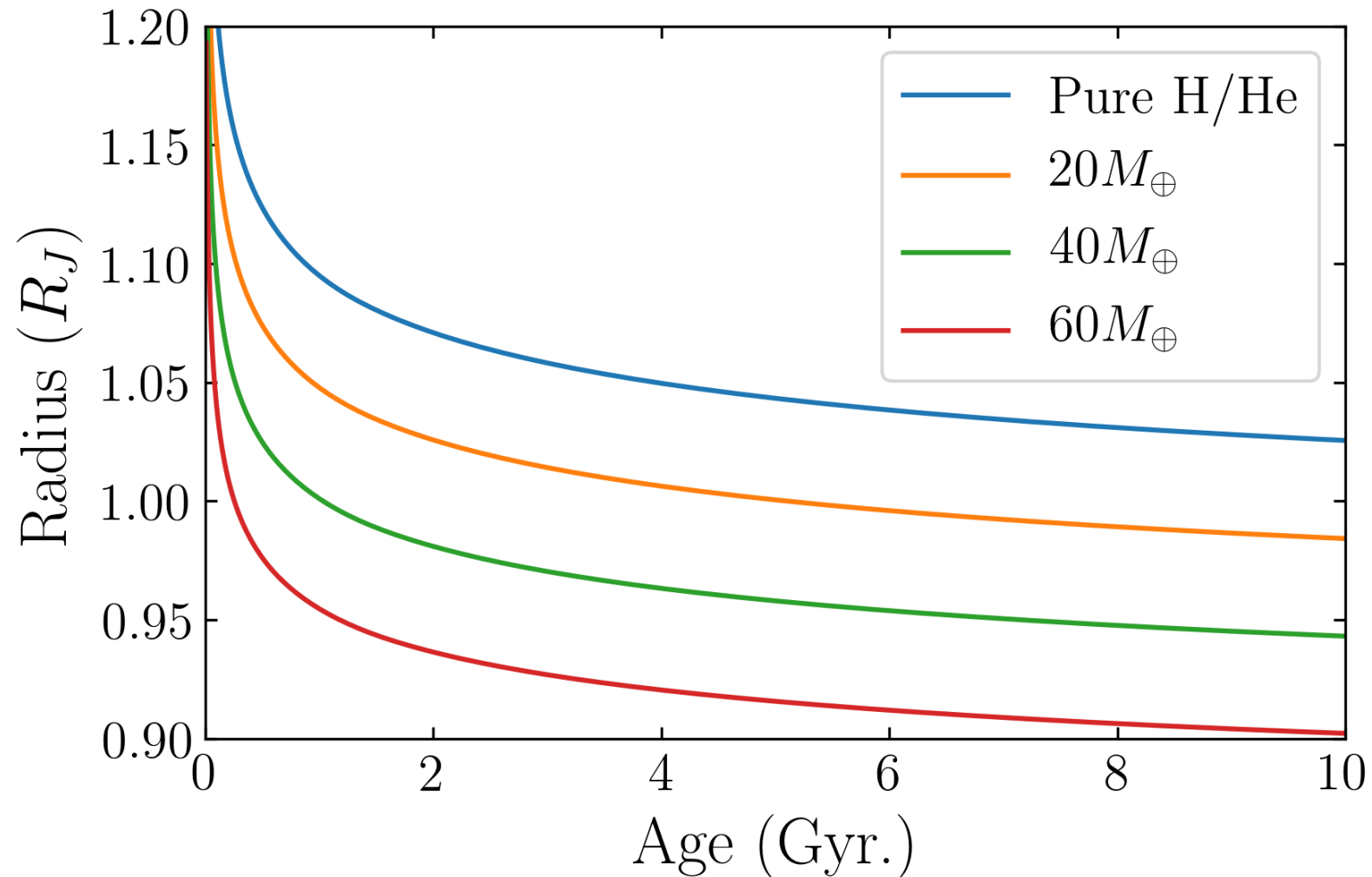


Thermal Evolution *via entropy*

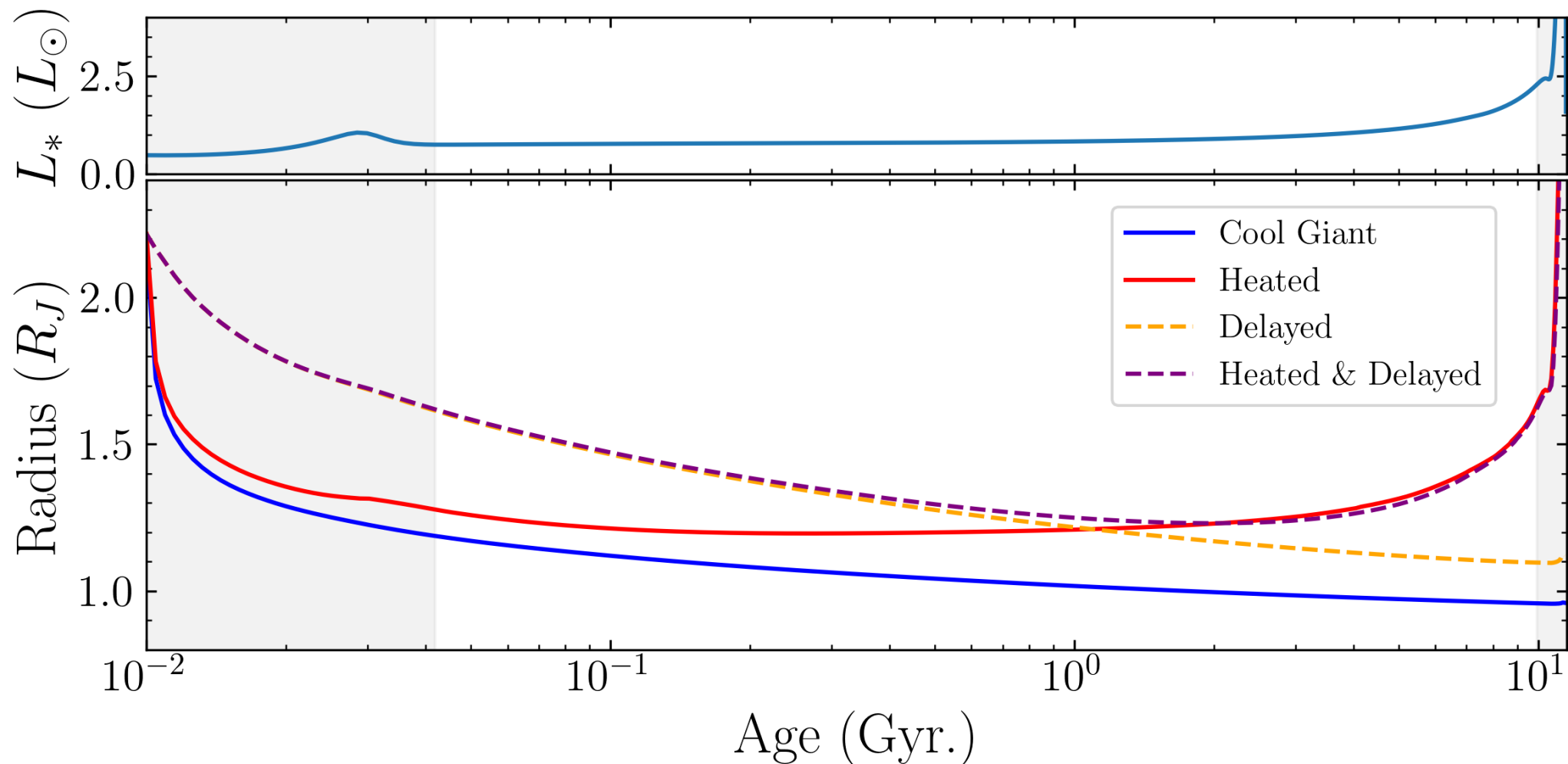
- Start from a large specific entropy (10-13 k_b / baryon)
- Calculate change in energy from: luminosity, hot Jupiter heating, deuterium burning, radioactivity, tides
- Convert to dt/ds , integrate

$$\frac{ds}{dt} = \frac{ds}{dE} \frac{dE}{dt} = \frac{dE}{dt} / \int_0^M T(m) dm$$

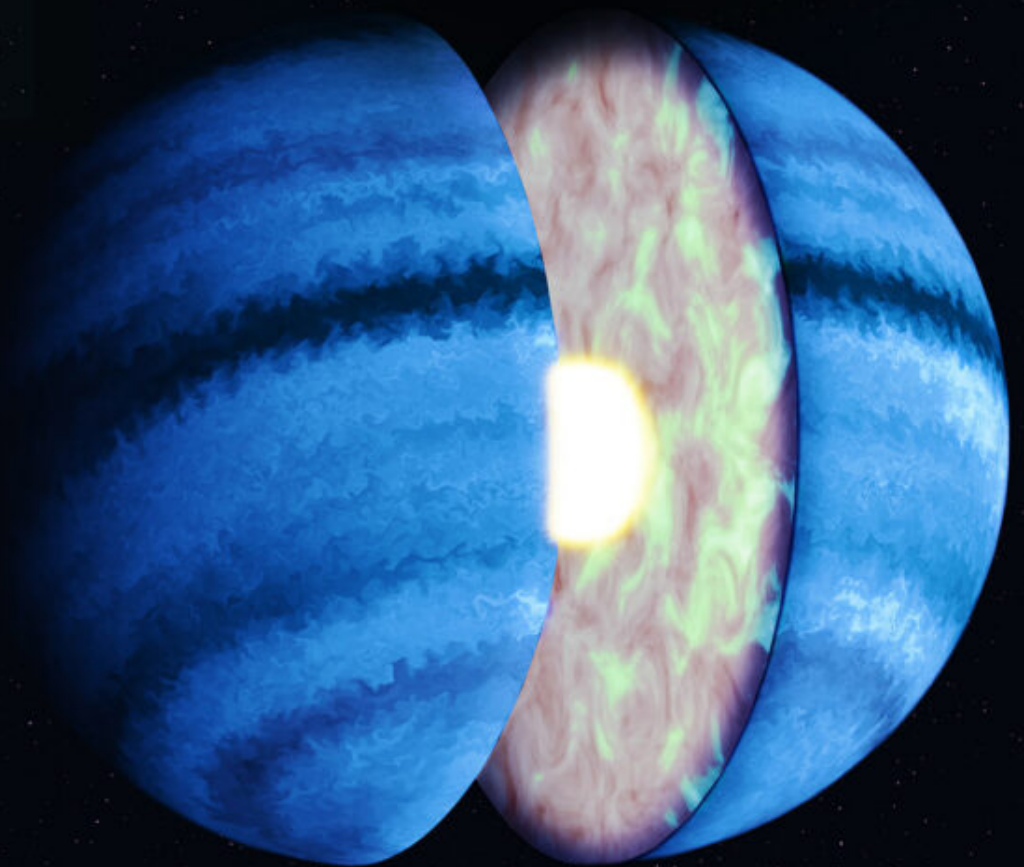
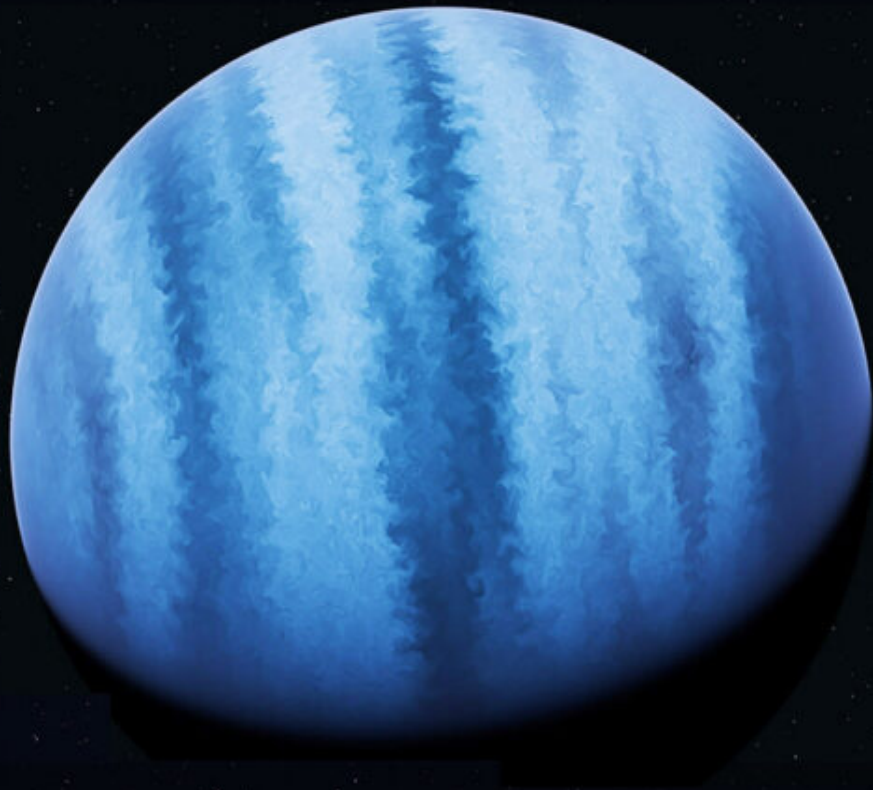
Warm Jupiter Evolution



Hot Jupiter Evolution



Current Areas of
investigation



Distorted Shapes

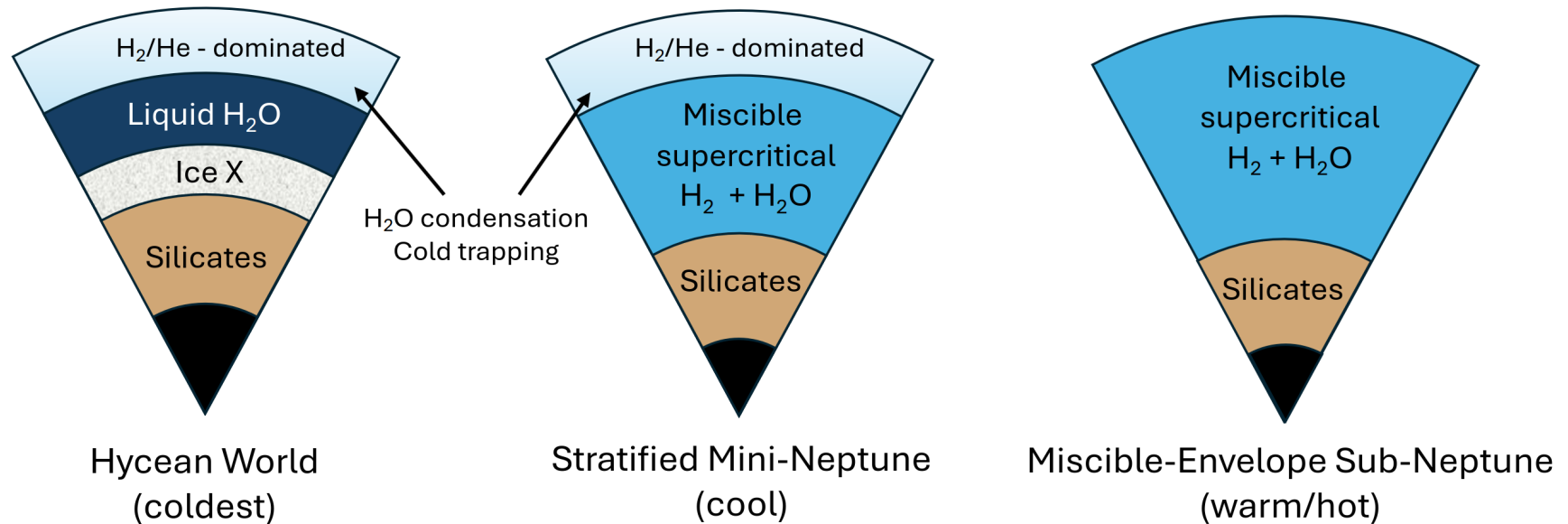
- Hot Jupiters can be prolate ellipsoids, cold Saturns oblate
- Observable (but hard) from transit, eclipse, and phase curves
- Occasionally can be inferred from dynamics (Batygin et al. 2009)
- Calculated from evolution models via Love numbers with:

$$r \frac{d\eta_2(r)}{dr} = 6 + \eta_2(r) - \eta_2(r)^2 - 6 \frac{\rho(r)}{\rho_m(r)} (\eta_2(r) + 1)$$

$$k_2 = \frac{3 - \eta_2(R)}{2 + \eta_2(R)}$$

See e.g. Love (1909), Leconte et al. (2011), Buhler et al. (2016), Akinsanmi et al. (2019), Hellard et al. (2019), Wahl et al. (2021), van Dijk & Miguel (2025)

Sub-Neptune Structures



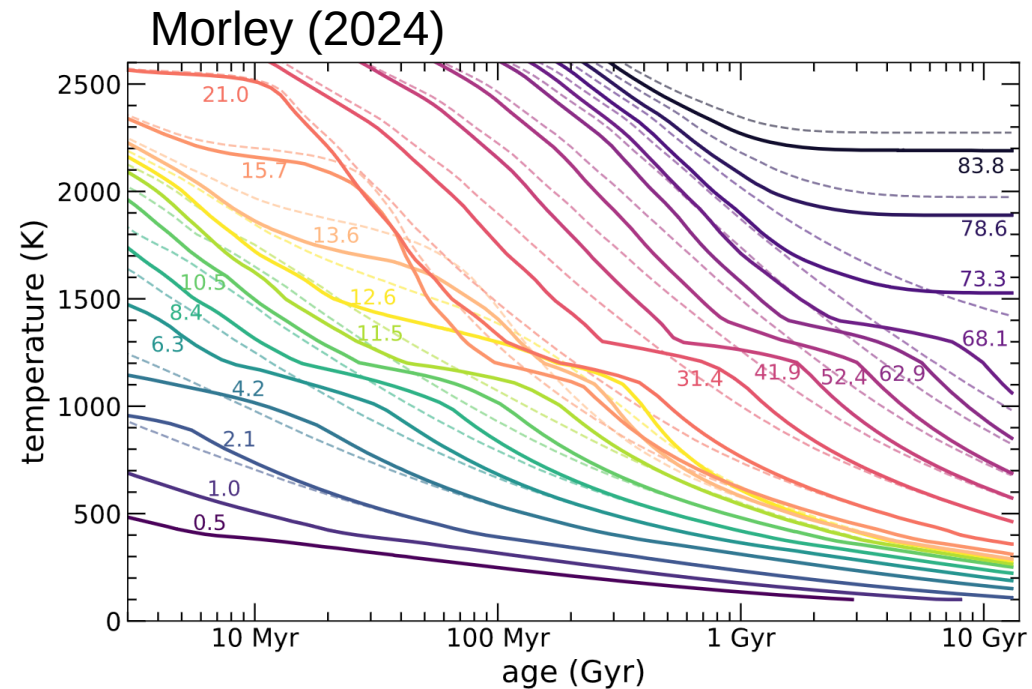
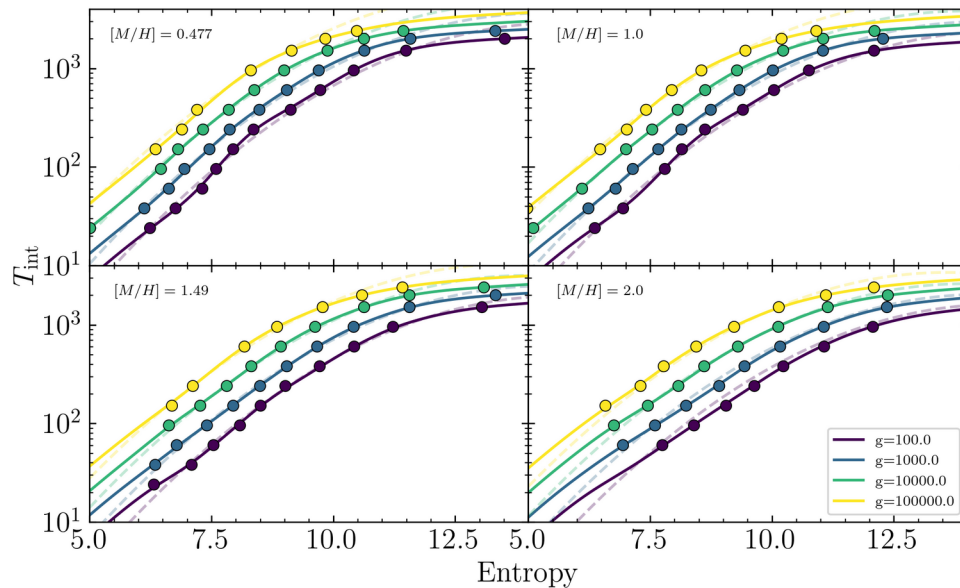
For essentially every sub-Neptune separately:

- Does the water form a separate layer (Benneke et al. 2019, Young 2025)?
- Can it support a liquid water ocean (Madhusudhan et al. 2020, Schmidt et al. 2025)?
- Is there a lava ocean at the base of the envelope?

Core Masses

- Can construct models with two metallicity parameters: core mass and envelope metallicity
- Tricky to constrain, need extra information:
 - Love number / gravity moments
 - Envelope metallicity (spectroscopy)
 - Evolution constraints
- See e.g. Bloot (2023), Sing (2025), van Dijk (2025)

Atmosphere Physics



Summary

- Can solve for interior structures efficiently when assuming adiabatic interiors
 - Hydrogen and Helium EOS most important, then water and other metals
 - Ambiguity on how metal should be structured
 - Evolution depends on atmosphere model and heat sources
- Sub-Neptune structure, chemistry are a major area of investigation
- Starting to infer core masses for exoplanets