

# Modelling Stars and Planets with Cesam2k20

Luke GAUVRIT<sup>1</sup>, Tristan GUILLOT<sup>1</sup>, Morgan DEAL<sup>2</sup>, Louis MANCHON<sup>3</sup>, João Pedro MARQUES<sup>4</sup>, Yveline LEBRETON<sup>3,5</sup>

<sup>1</sup> Laboratoire Lagrange, Univ. Côte d'Azur • <sup>2</sup> LUPM, Univ. Montpellier • <sup>3</sup> LIRA, Obs. de Paris, Univ. Paris-Saclay  
• <sup>4</sup> IAS, Univ. Paris-Saclay • <sup>5</sup> IPR, Univ. Rennes



# I. Motivation and Context

## i. Why Extend Cesam2k20 to Exoplanets?

### Code d'Evolution Stellaire Adaptatif et Modulaire

- B-spline collocation solver: high-order, smooth, accurate
- Adaptive meshes: structure, diffusion, rotation on grids
- 1D stellar evolution: PMS to He-burning

*Manchon et al. (2025)*

Cesam2k20 provides the initial PLATO stellar grids

#### Stellar Observables

- Photometry
- Spectroscopy
- Asteroseismology
- etc...

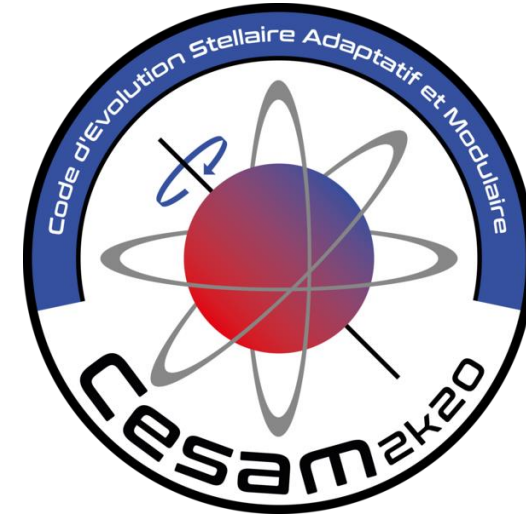
CESAM's



grid

#### Stellar Parameters

- Mass
- Radius
- Age
- Composition
- etc...

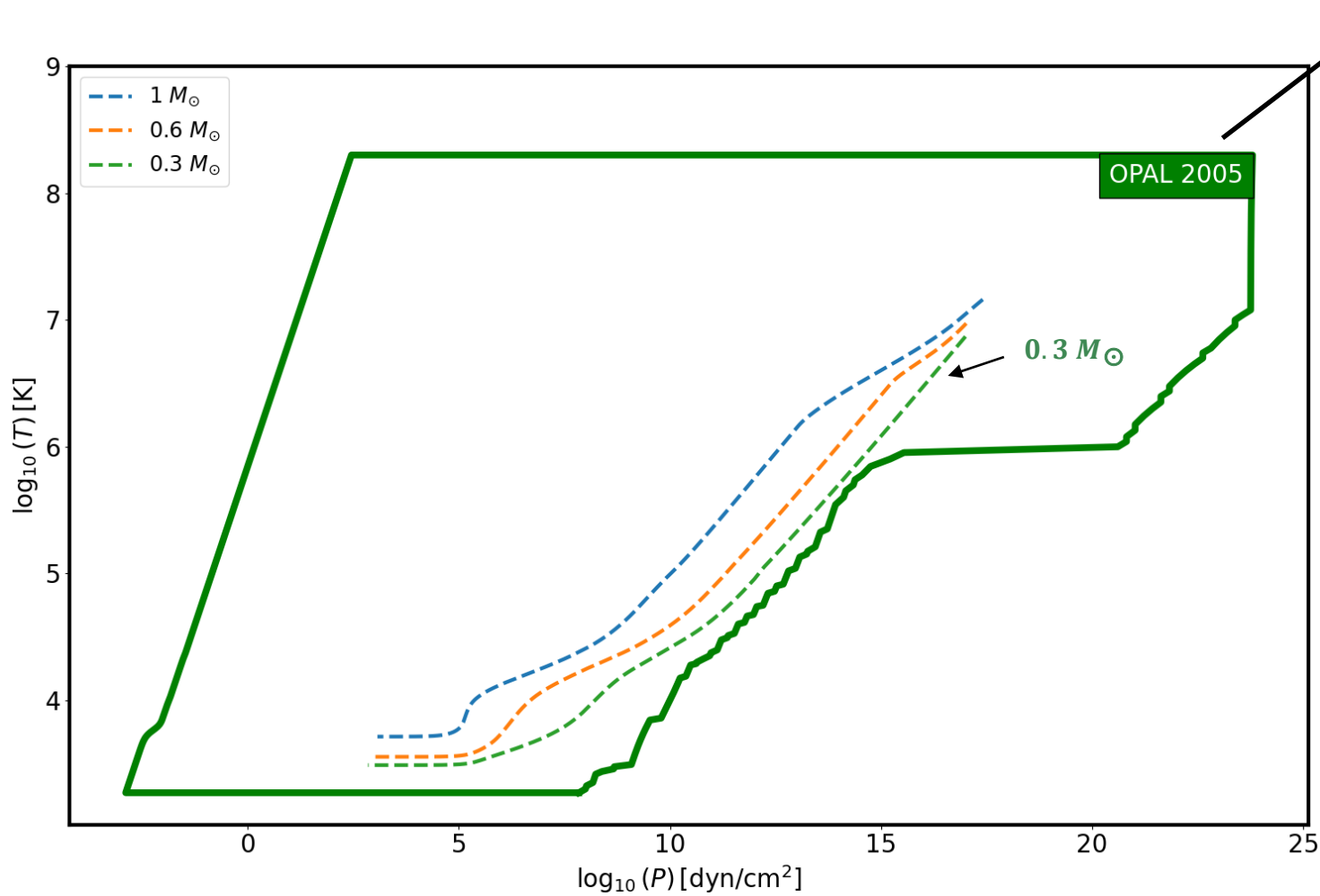


### Extend Cesam2k20 to Study Exoplanets

- Aligns with PLATO pipelines (already used for stars)
- Ensures consistent physics for stars and planets
- Avoids dual-code workflows → better integration

# I. Motivation and Context

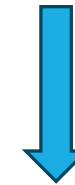
## ii. Strategy: Using New Equation of State Tables



Rogers & Nayfonov (2002)

## Equation of State limitations

- OPAL2005 is the main EoS table of CESAM2k20
  - Limited Pressure-Temperature range
  - Lowest reachable mass:  $0.3 M_{\odot}$



**New EoS tables with a wider P-T range**

# I. Motivation and Context

## ii. Strategy: Using New Equation of State Tables

### The **equation of state** (EoS) in physics:

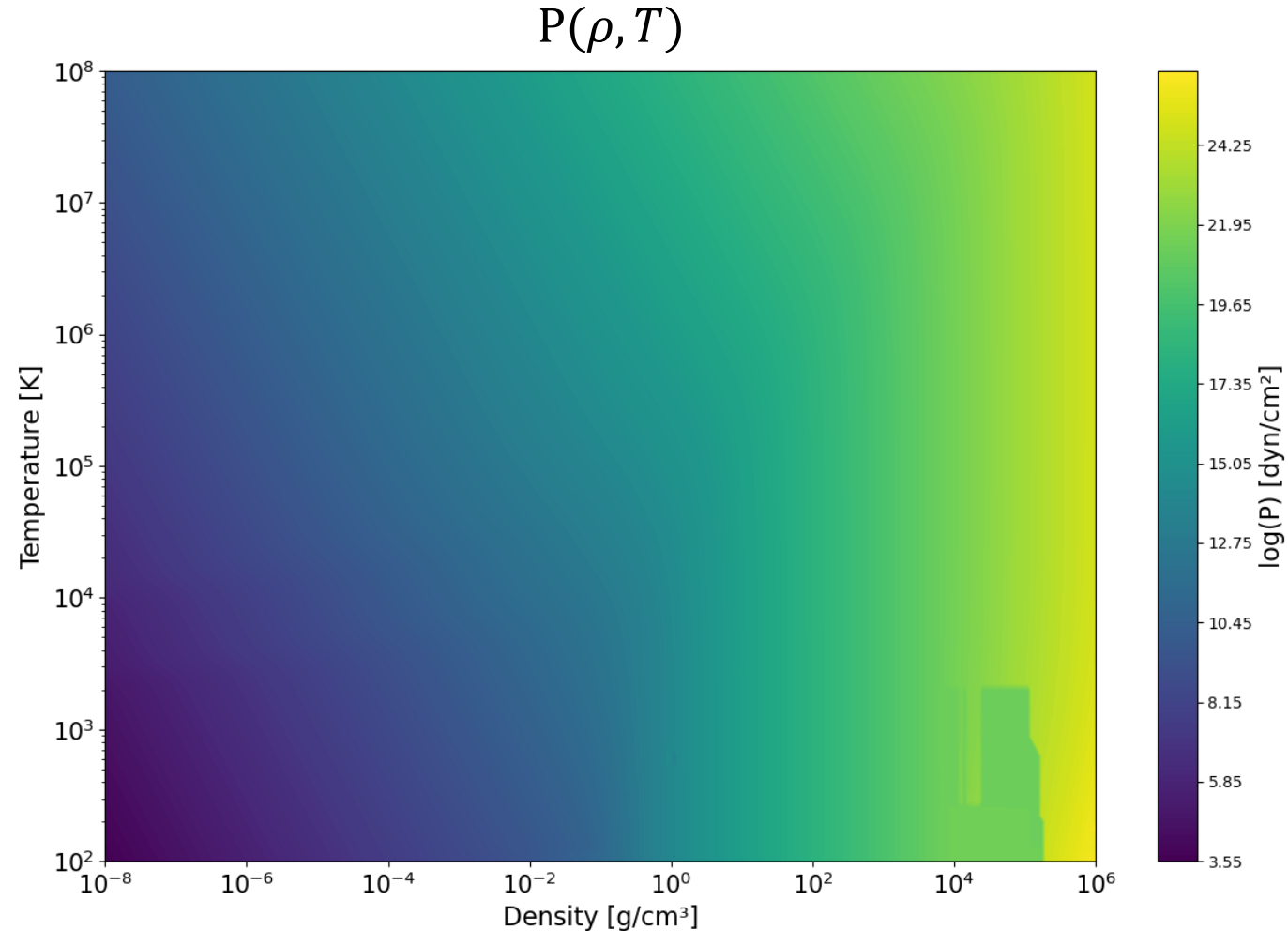
- Bridge between thermodynamics and structure

Built using **Numerical Simulations**:

Chemical Picture	Physical Picture
Free-energy minimization	DFT-MD, QMD, PIMC, ...
Saumon, Chabrier & van Horn (1995) Hummer & Mihalas (1998)	Chabrier & Potekhin (1998) Rogers & Nayfonov (2002) Militzer & Hubbard (2013)

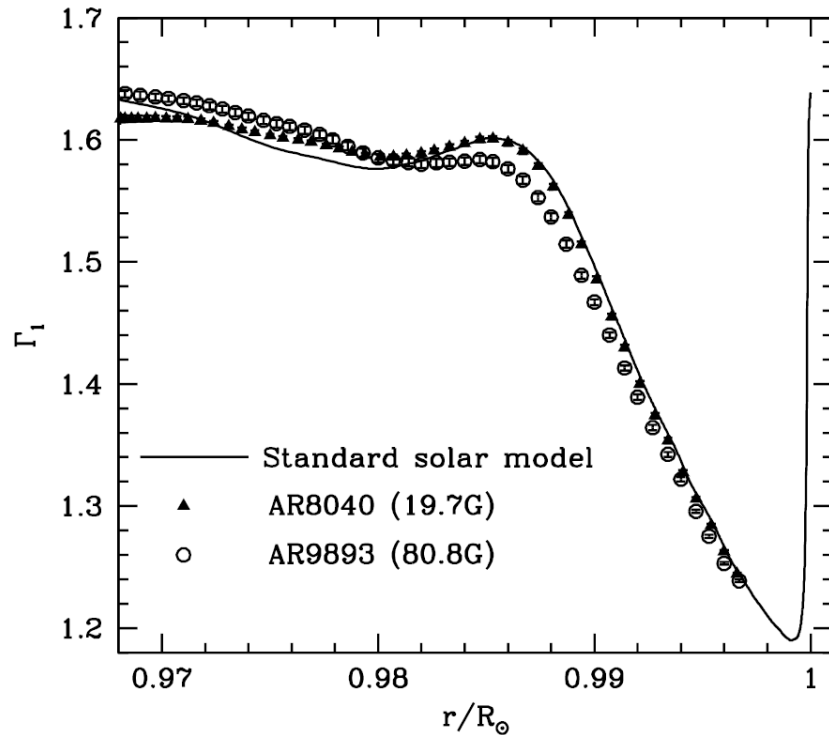
We will use three: **CMS19** (*Chabrier et al. 2019*)

*MH13* { **CD21** (*Chabrier & Debras 2021*)  
**HG23** (*Howard & Guillot 2023*)

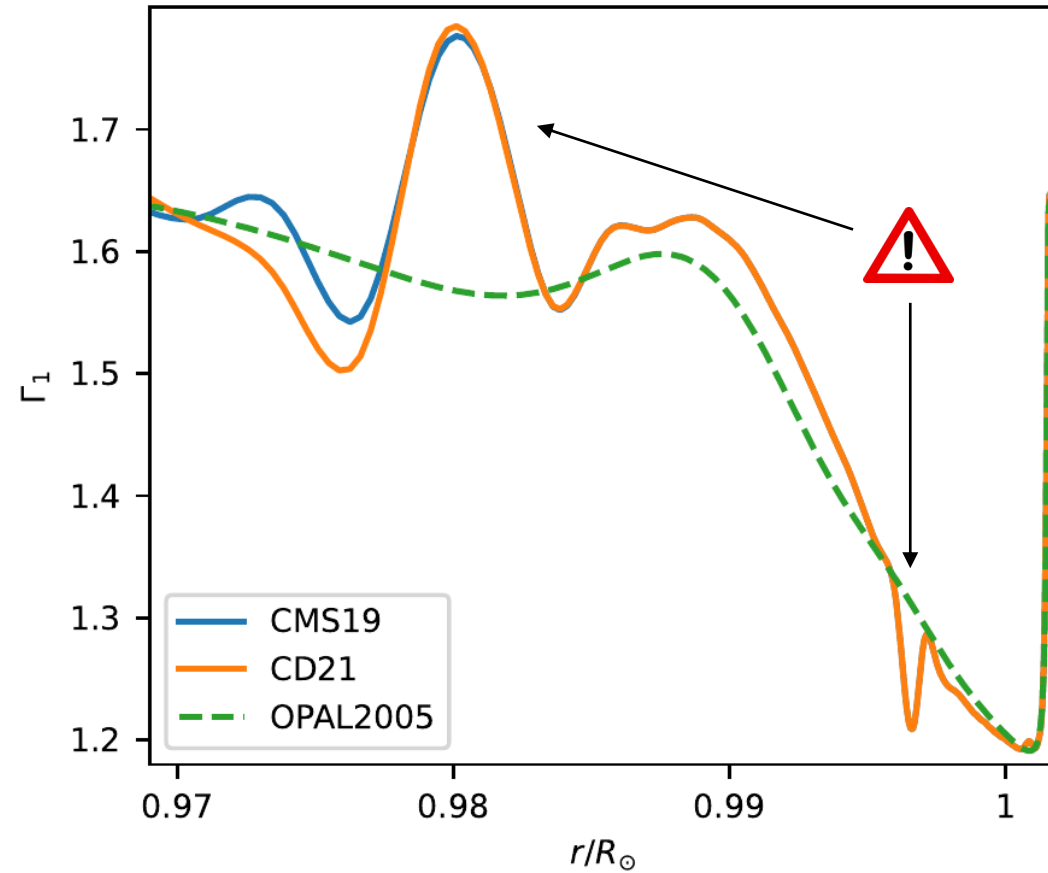


# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

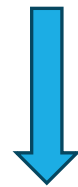


Taken from Basu et al. (2004)



$$\Gamma_1 = \left( \frac{\partial P}{\partial \rho} \right)_S$$

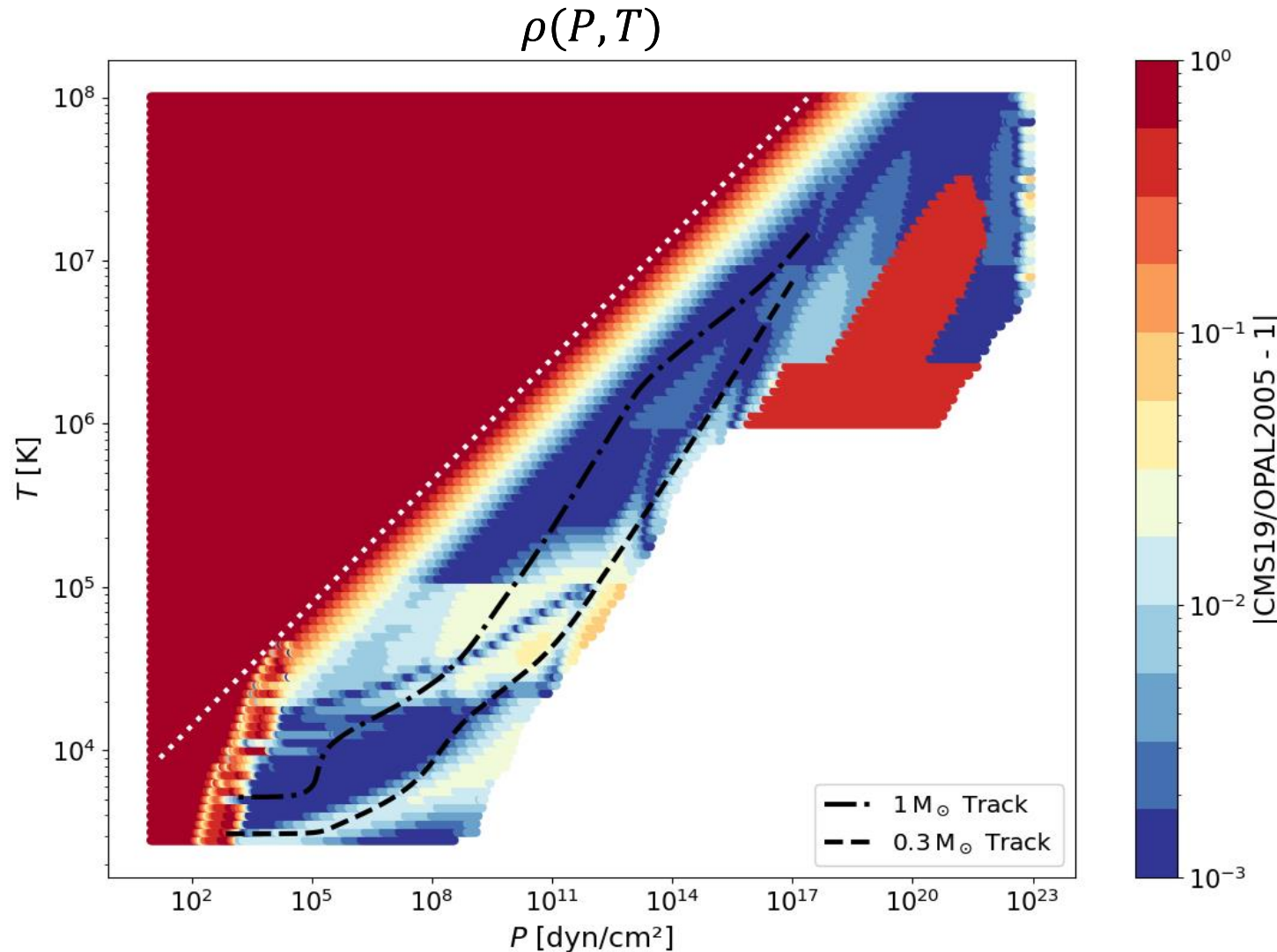
OPAL2005 aligned with observations, CMS19 and CD21 do not



**CMS19 and CD21 are incompatible with asteroseismic constraints**

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

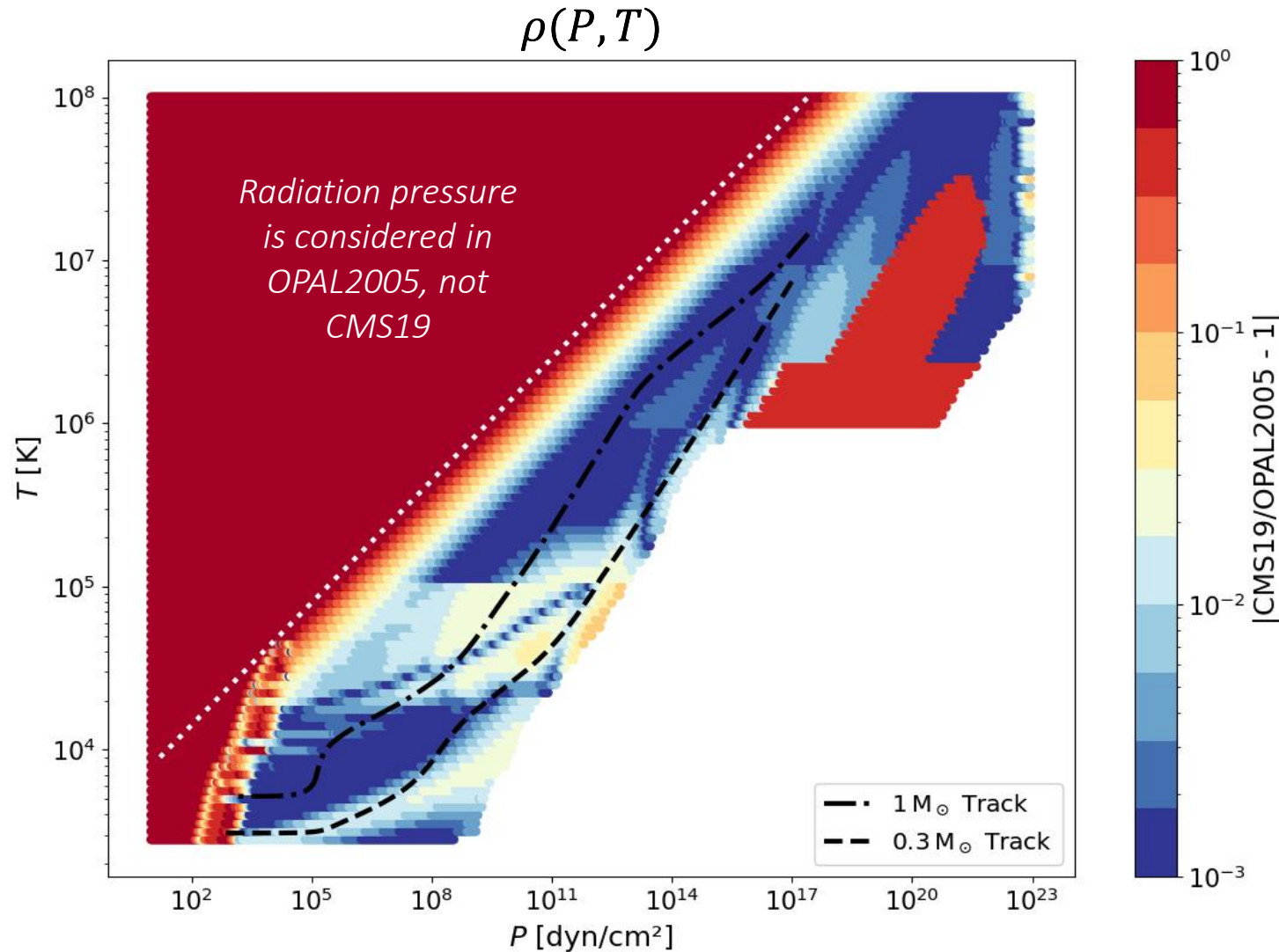


- Scatter plot  $\longrightarrow$  each point is a grid point of CMS19.
- Computed the density outputs of the OPAL2005 and CMS19 EoS routines for all PT points in the OPAL2005 validity domain.
- Relative differences computed as:

$$\left| \frac{\text{CMS19}}{\text{OPAL2005}} - 1 \right|$$

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

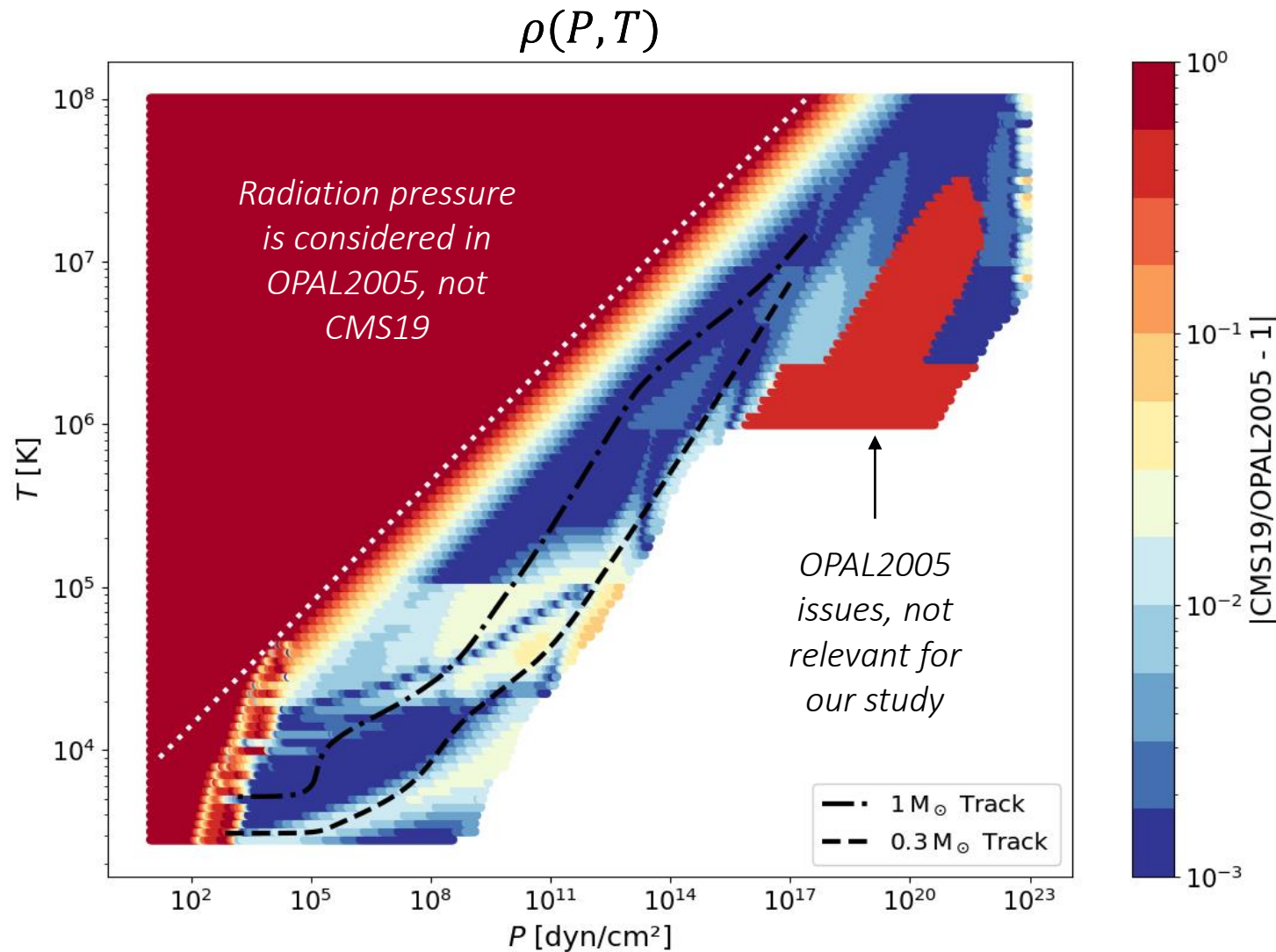


- White dotted-line: **radiation pressure**

$$P_{\text{rad}}(T) = a \frac{T^4}{3}$$

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models



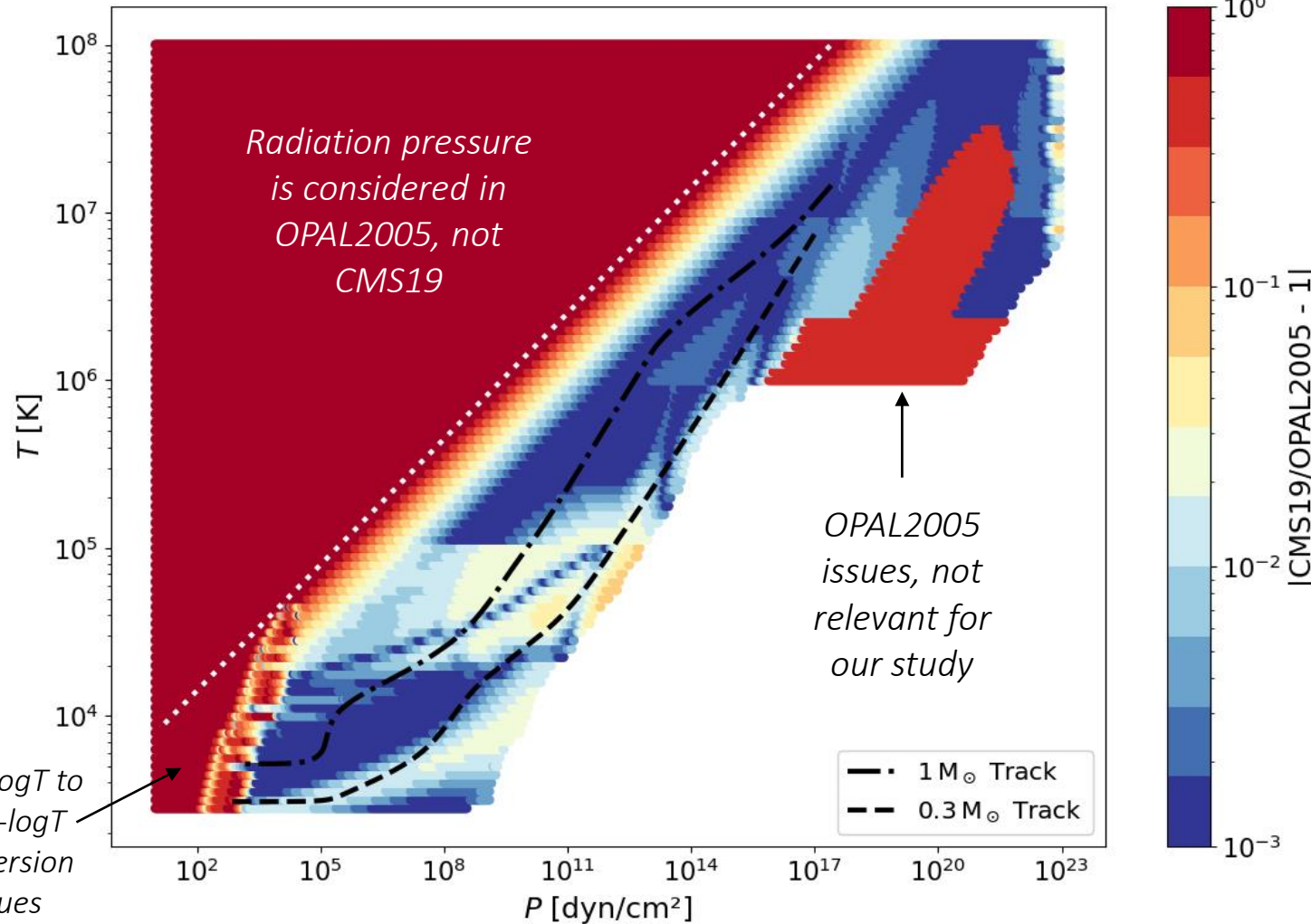
- White dotted-line: **radiation pressure**

$$P_{\text{rad}}(T) = a \frac{T^4}{3}$$

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

$\rho(P, T)$



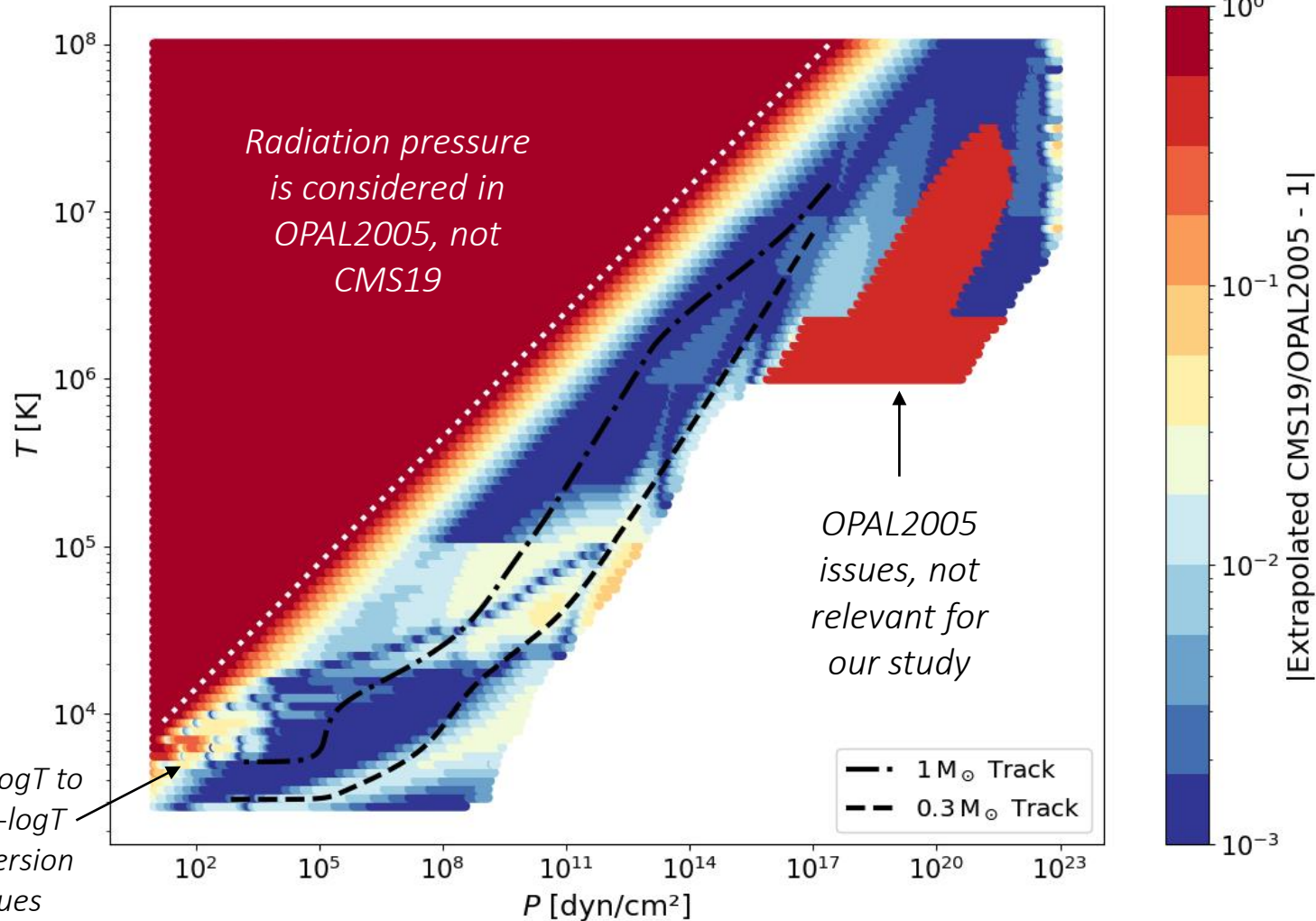
- White dotted-line: **radiation pressure**

$$P_{\text{rad}}(T) = a \frac{T^4}{3}$$

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

$$\rho(P, T)$$



- **Extrapolation** improves OPAL-CMS19 overlap agreement

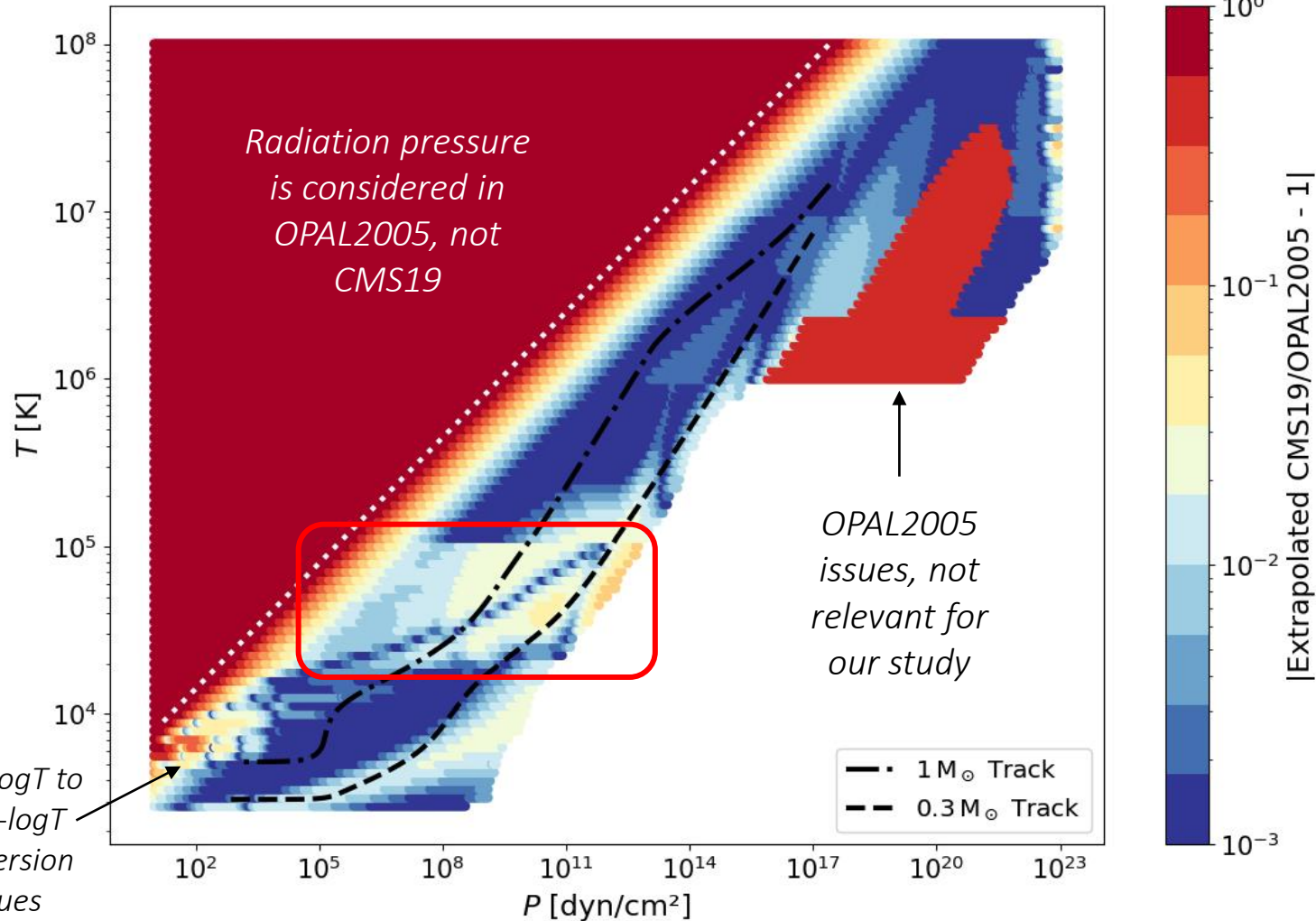
$$T_0 = T \left( \frac{P}{P_0} \right)^{-0.07}$$

- But low-T domain cannot be **validated** for the brown dwarf/planetary regime

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

$$\rho(P, T)$$



- **Extrapolation** improves OPAL-CMS19 overlap agreement

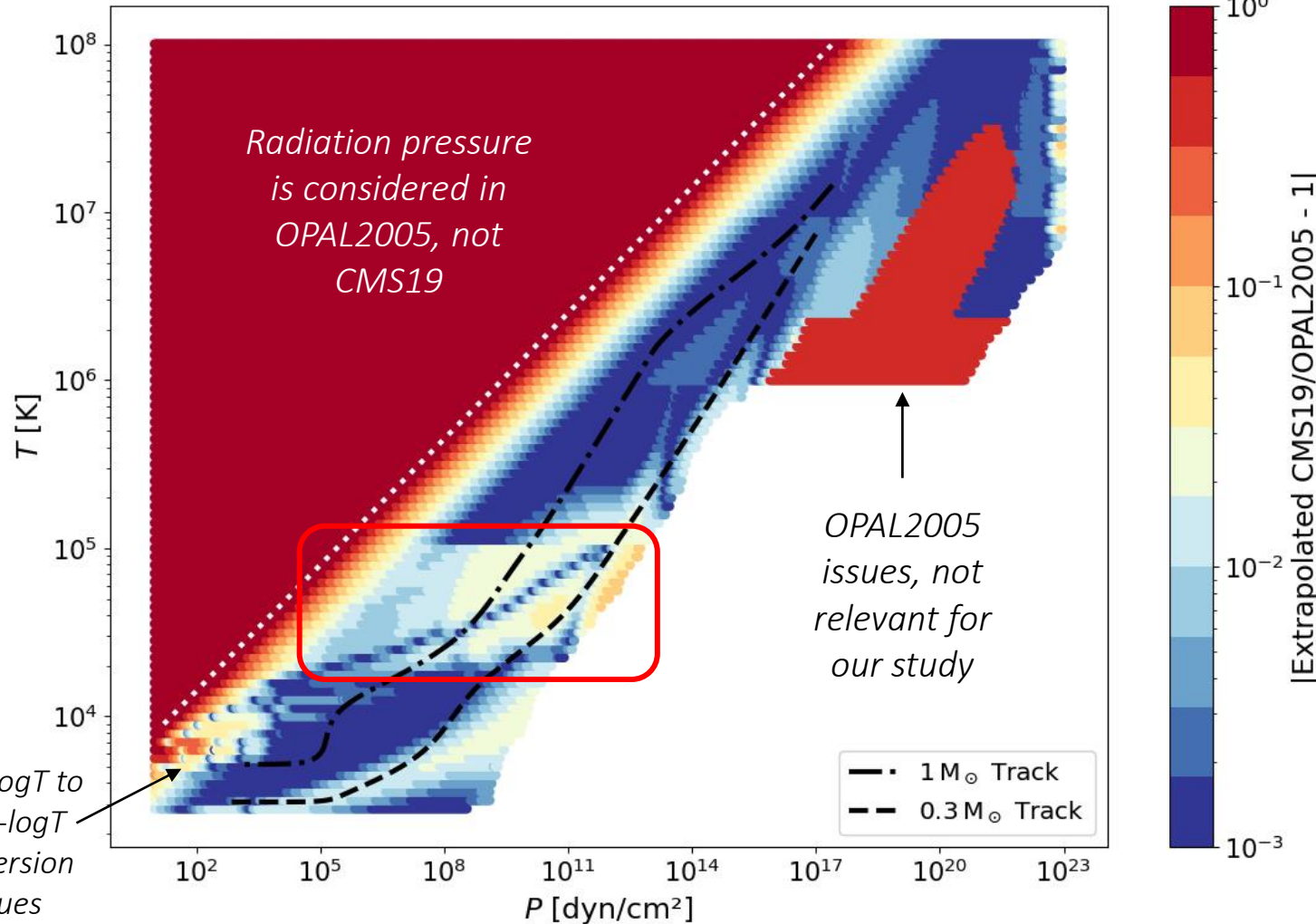
$$T_0 = T \left( \frac{P}{P_0} \right)^{-0.07}$$

- But low-T domain cannot be **validated** for the brown dwarf/planetary regime

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models

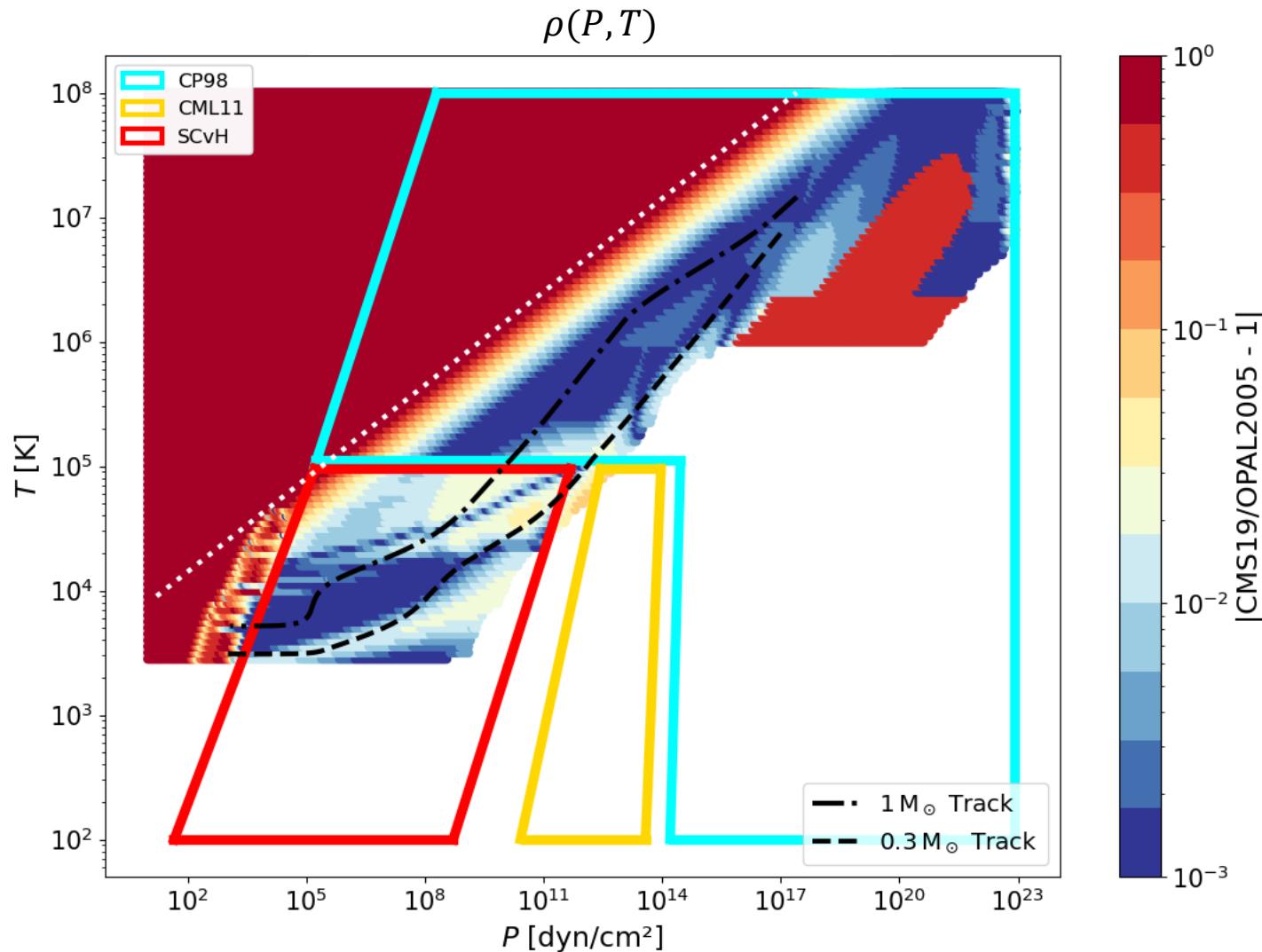
$$\rho(P, T)$$



**CMS19 is tuned for planets: invalid for stellar modelling**

# II. Discrepancies With the Equation of State

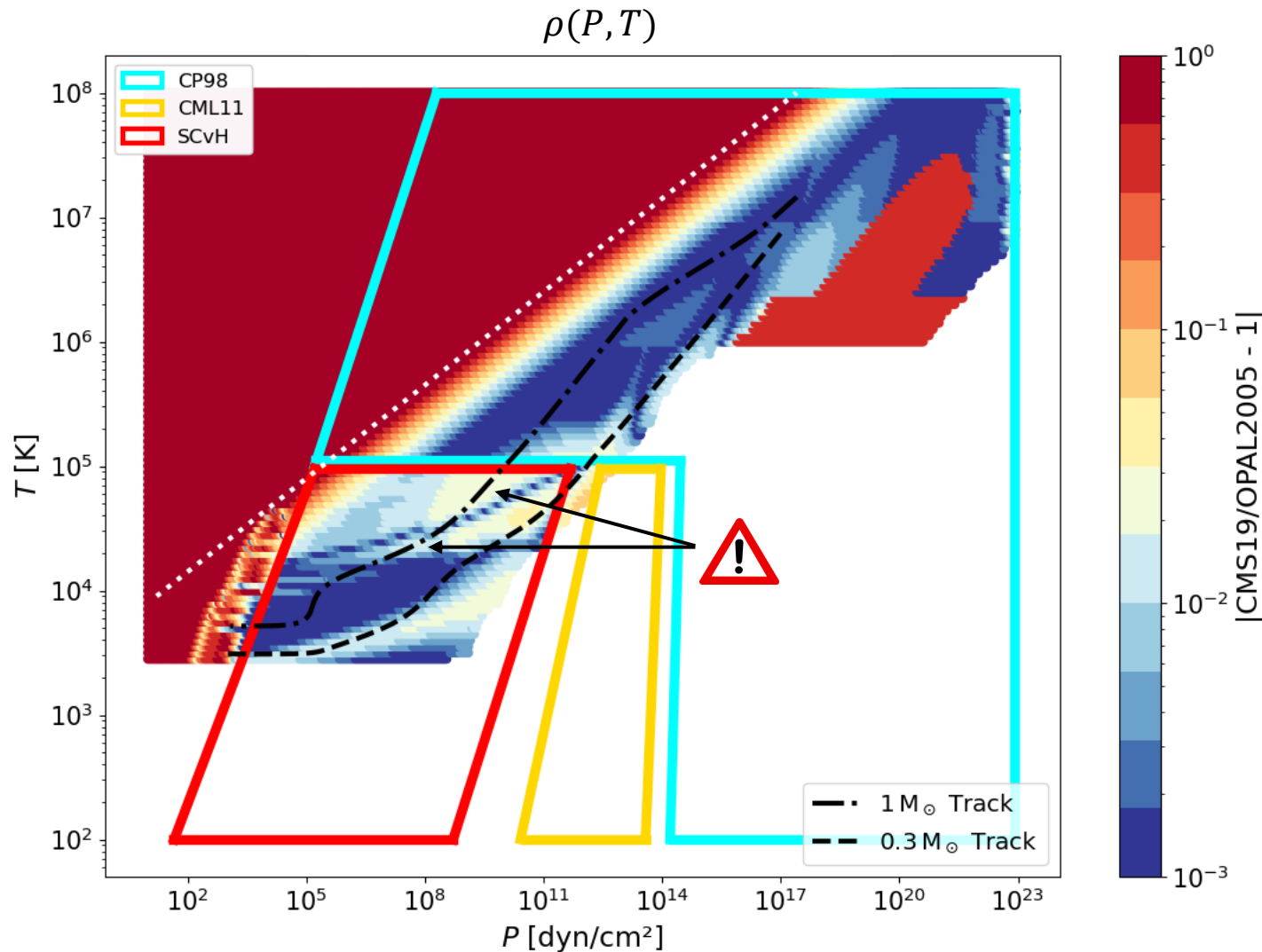
## i. A Problem With Solar Models



$$\text{CMS19} = \text{SCvH} + \text{CML11} + \text{CP98}$$

# II. Discrepancies With the Equation of State

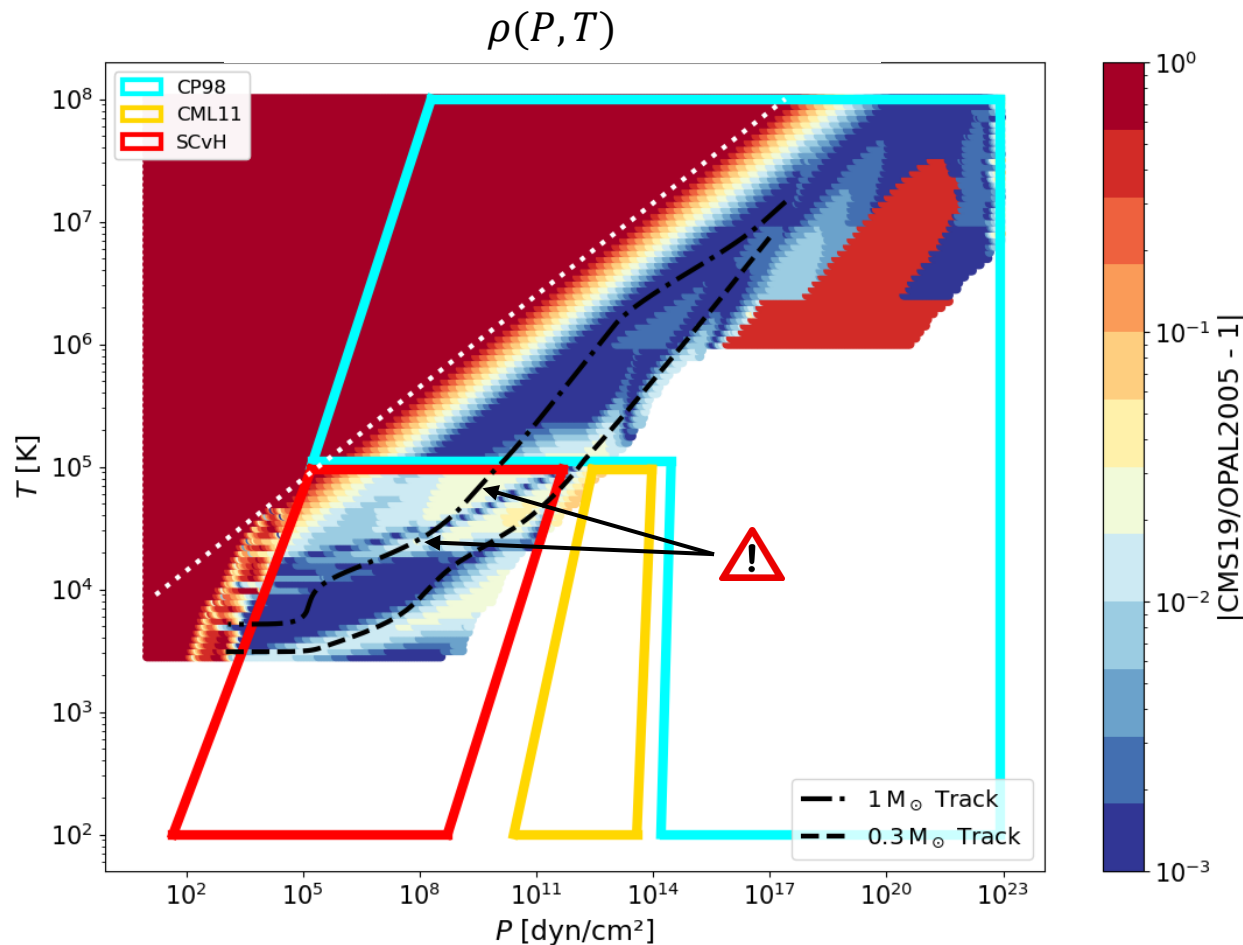
## i. A Problem With Solar Models



$$\begin{aligned} & \text{CMS19} \\ & = \\ & \text{SCvH} \\ & + \\ & \text{CML11} \\ & + \\ & \text{CP98} \end{aligned}$$

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models



Taken from Saumon et al. (1995)

“ This microfield acts as a time-dependent perturbation on the Coulomb potential of the nucleus and can induce Stark ionization of the upper levels of an atom.

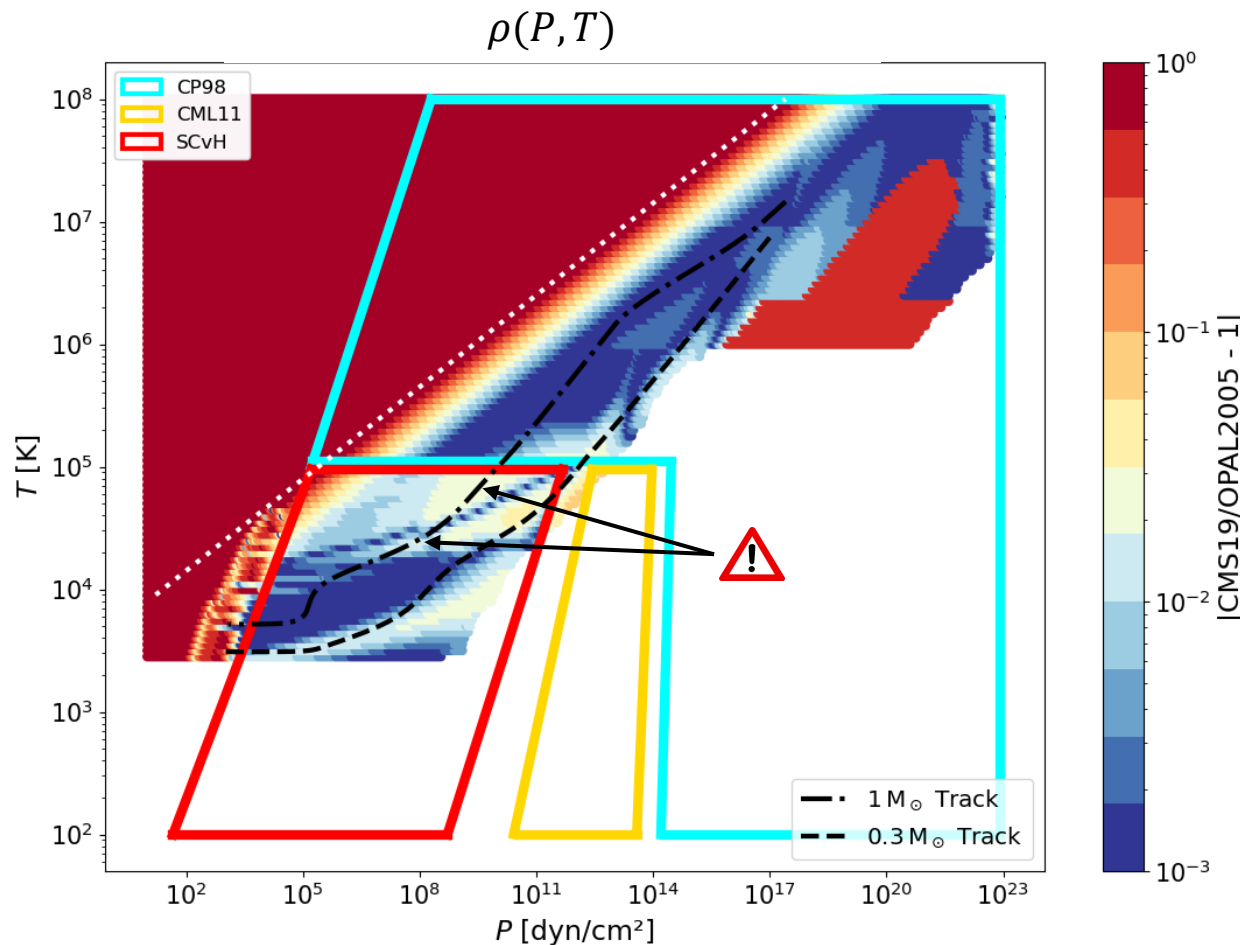
...

Recall that the SC EOS does not include microfield effects, so that the MHD EOS is superior for solar model calculations. ”

**SCvH neglects microfield effects**, leading to inaccurate ionization and too many bound states at higher density

# II. Discrepancies With the Equation of State

## i. A Problem With Solar Models



Taken from Saumon et al. (1995)

“ This microfield acts as a time-dependent perturbation on the Coulomb potential of the nucleus and can induce Stark ionization of the upper levels of an atom.

...

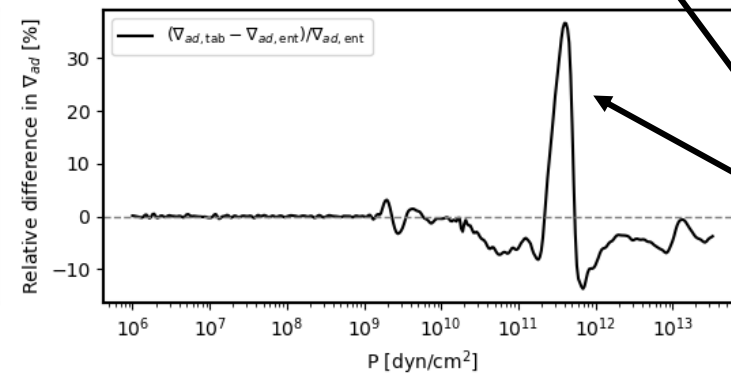
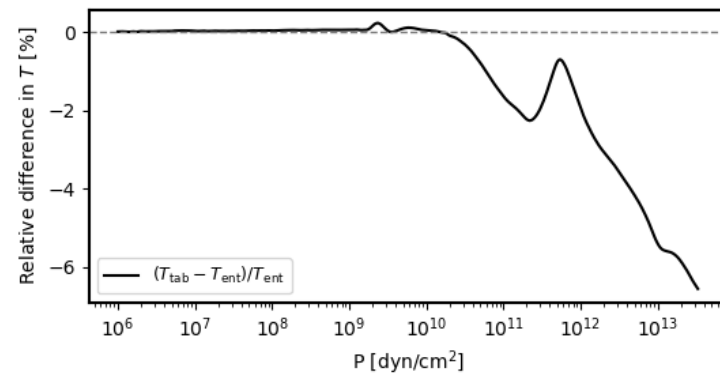
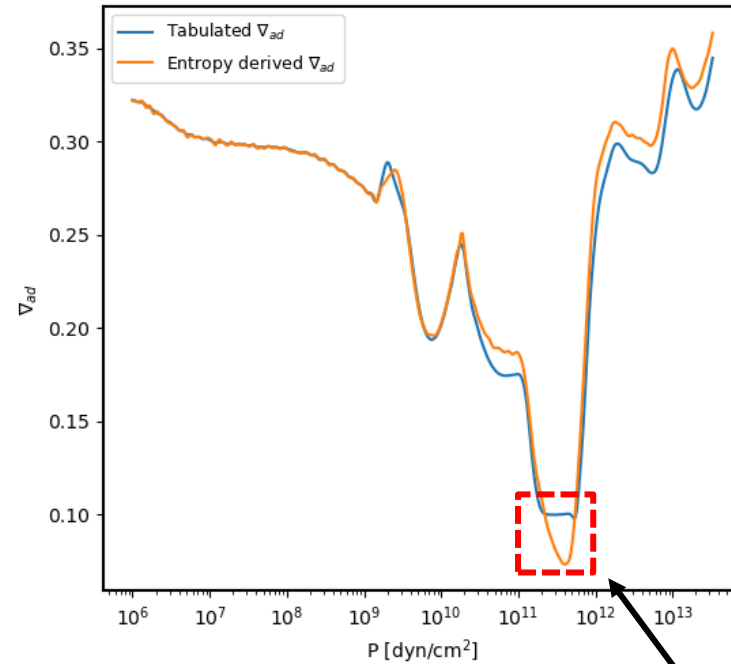
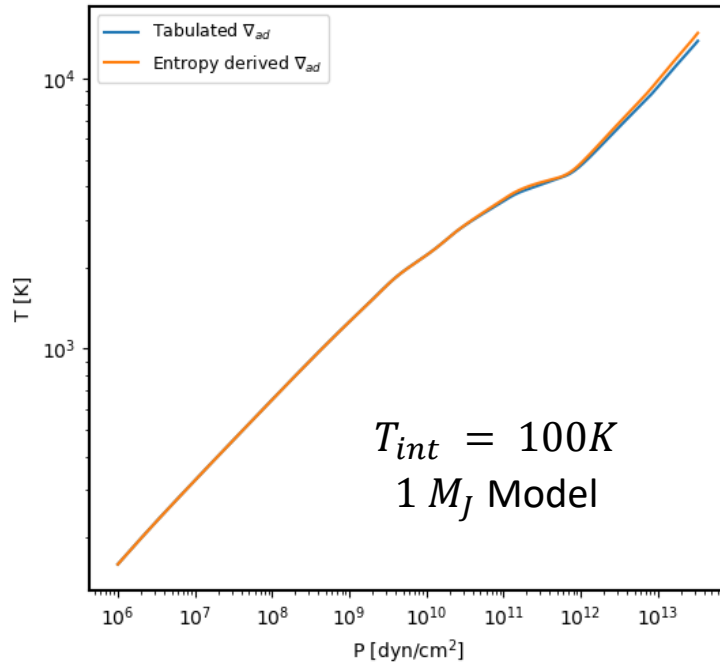
Recall that the SC EOS does not include microfield effects, so that the MHD EOS is superior for solar model calculations. ”

**SCvH neglects microfield effects**, leading to inaccurate ionization and too many bound states at higher density

Issue with SCvH at higher P,T + OPAL has a lower pressure boundary → OPAL/CMS merger?

# II. Discrepancies With the Equation of State

## ii. A Problem With Planetary Models



EoS Table used: **CD21**

$$\nabla_{ad} = \left( \frac{\partial \ln T}{\partial \ln P} \right)_S$$

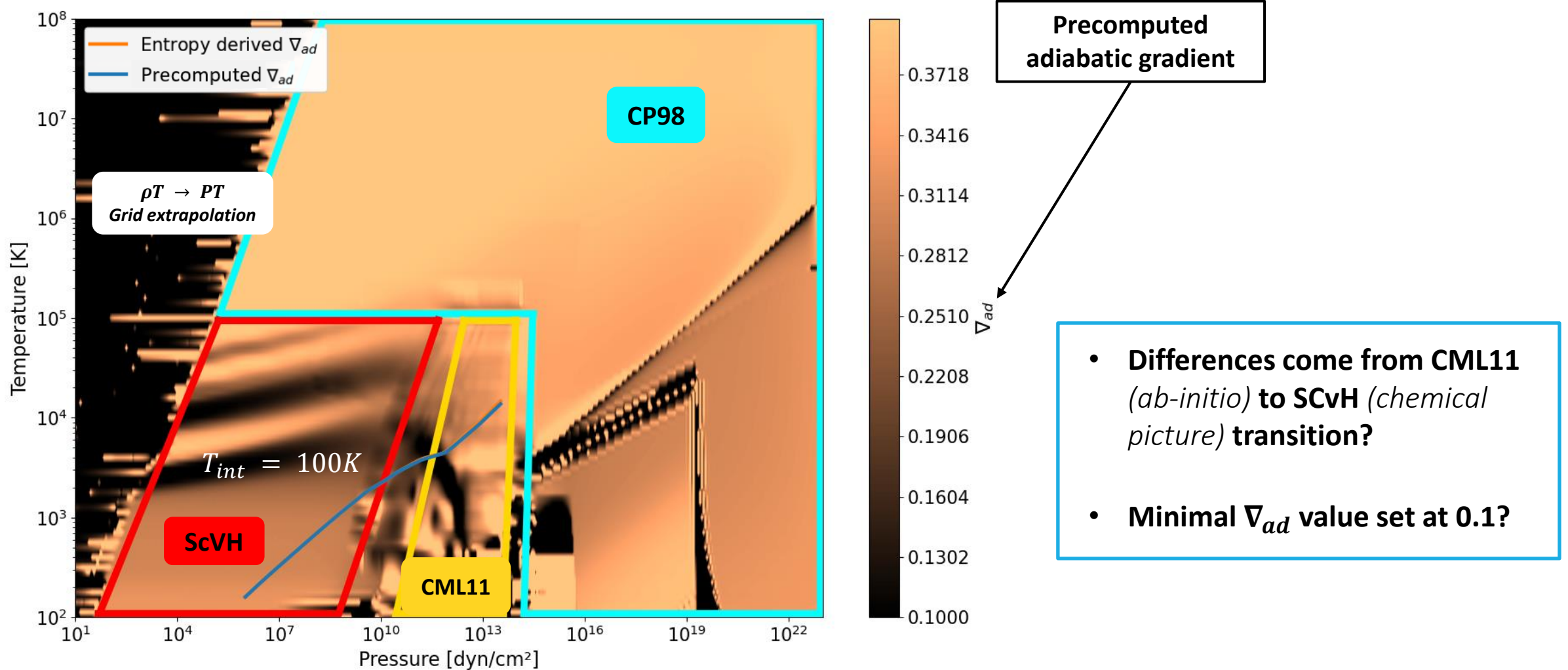


$$\nabla_{ad} = - \frac{P}{T} \frac{\left( \frac{\partial S}{\partial P} \right)_T}{\left( \frac{\partial S}{\partial T} \right)_P}$$



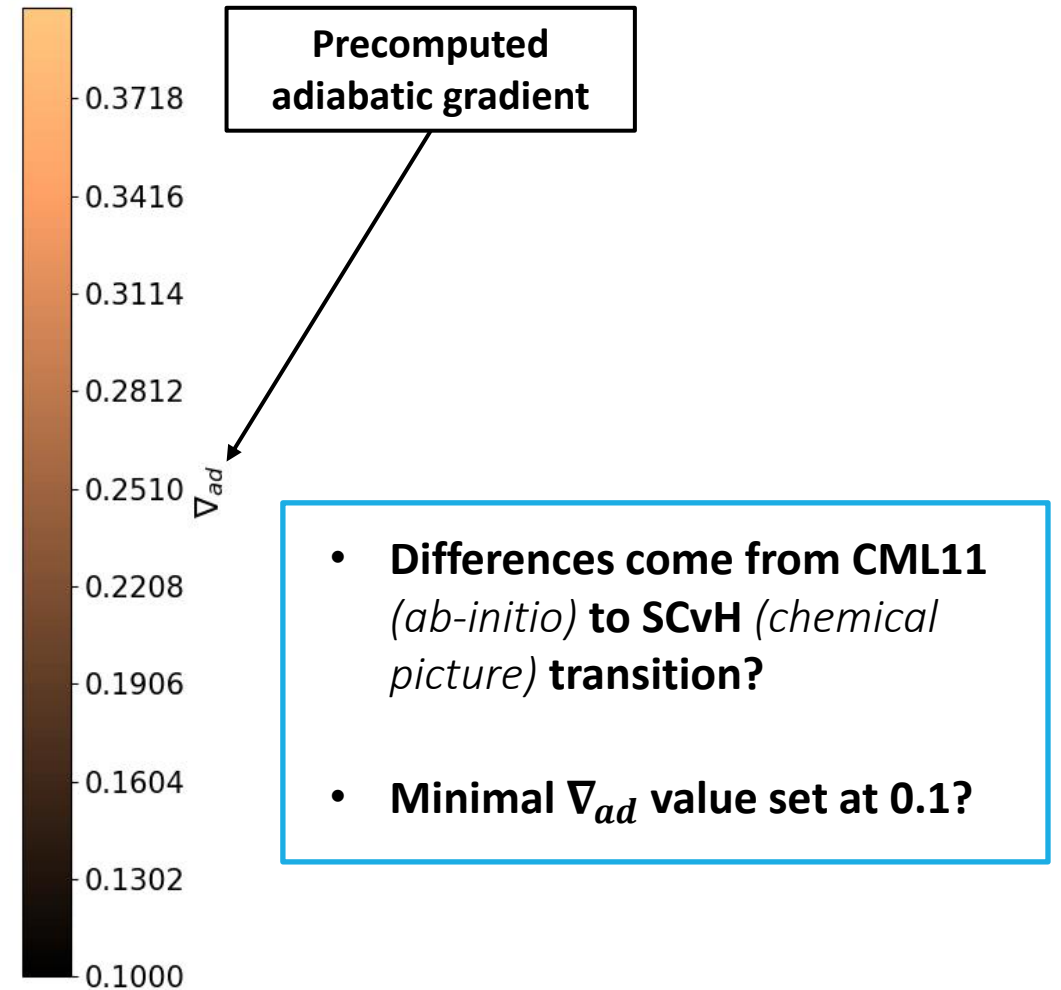
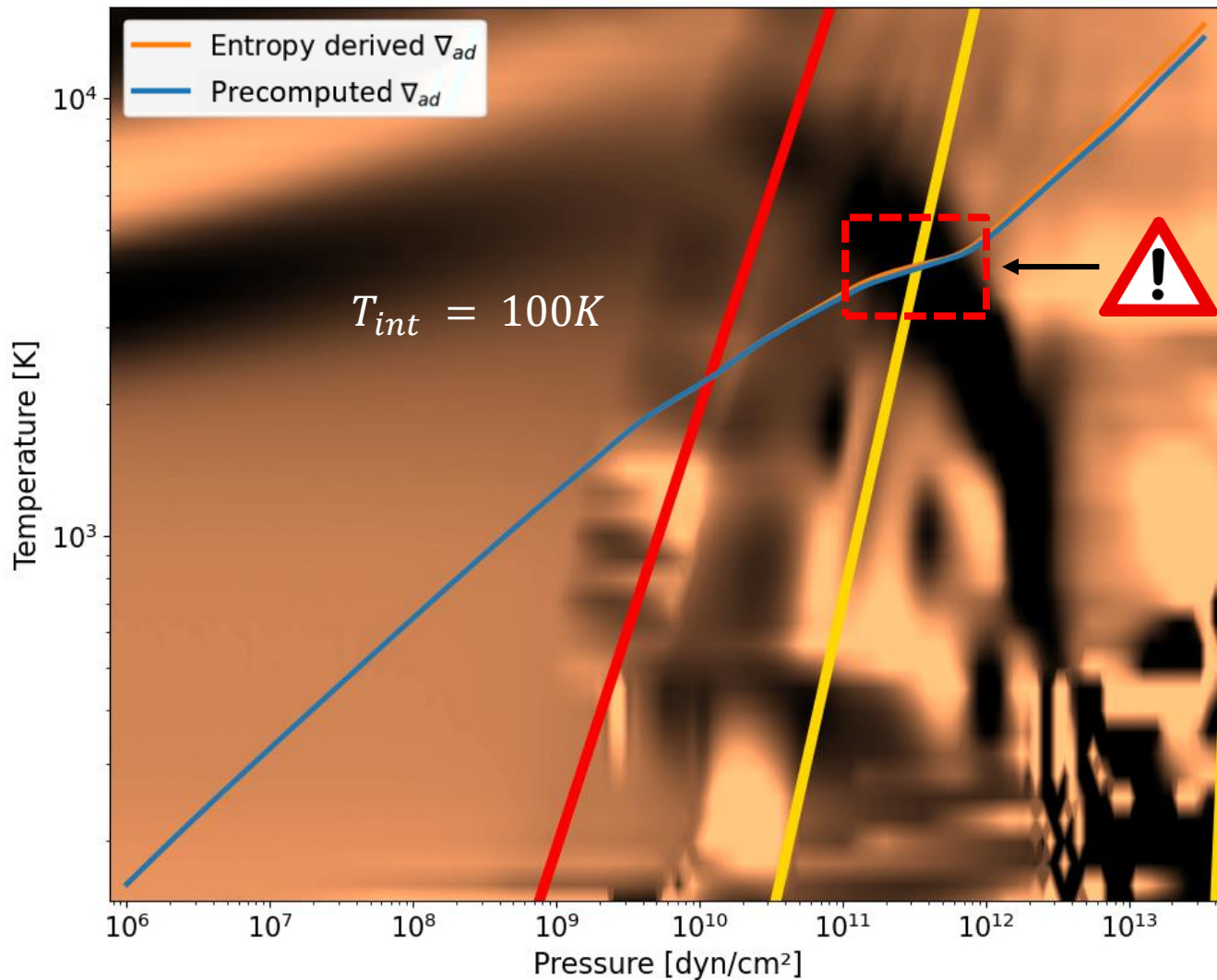
# II. Discrepancies With the Equation of State

## ii. A Problem With Planetary Models



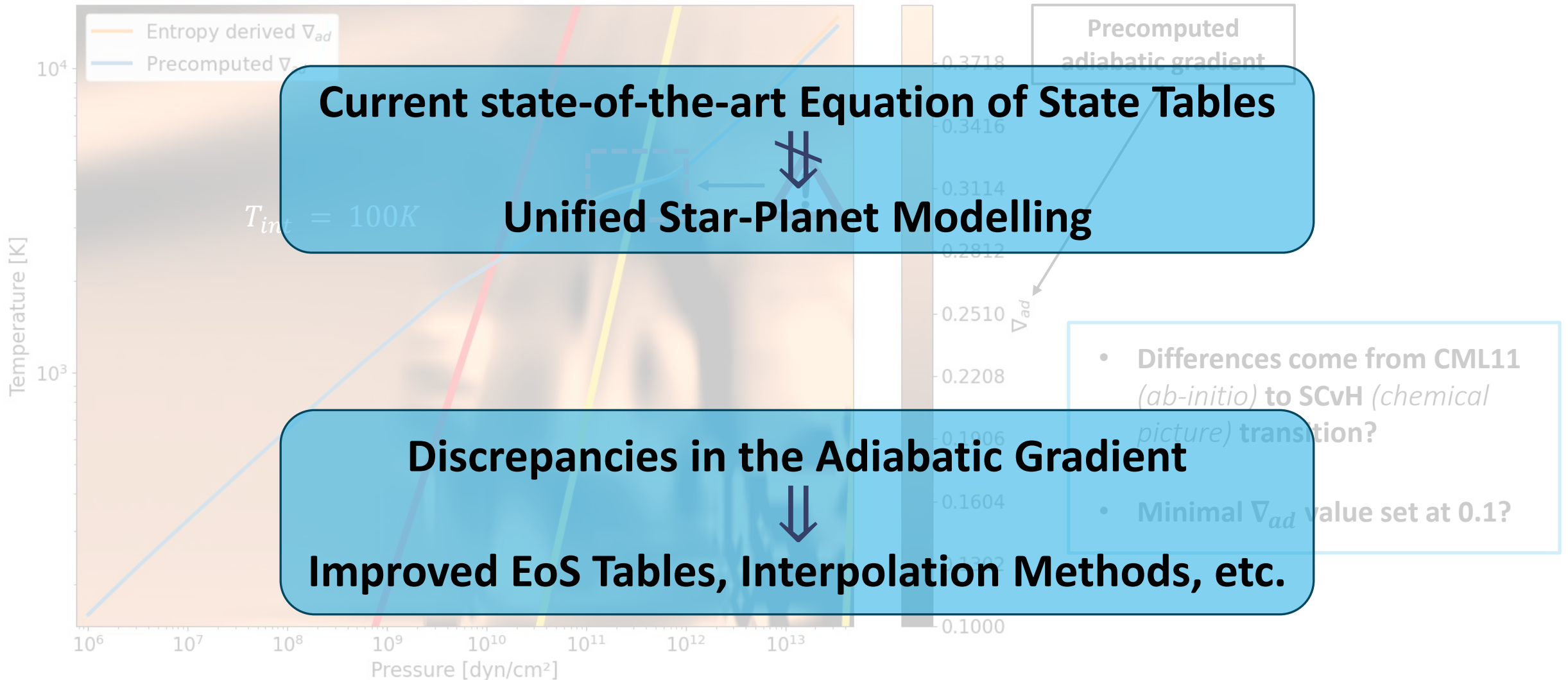
# II. Discrepancies With the Equation of State

## ii. A Problem With Planetary Models



# II. Discrepancies With the Equation of State

## ii. A Problem With Planetary Models



# Modelling Stars and Planets With CEsam2k20

## i. Planetary EoS Tables: a Brief Overview

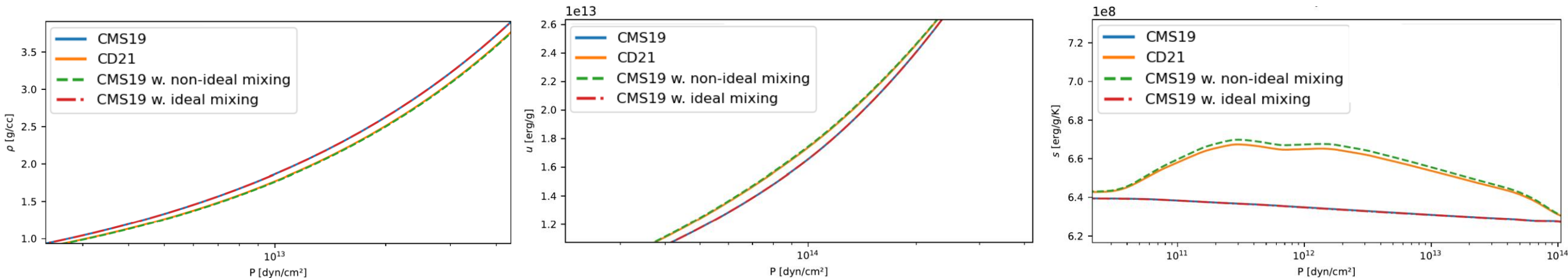
Differences between CMS19 and CD21  $\rightarrow$  **Non-Ideal Mixing Effects due to H/He interactions** (Based on MH13)

$$\frac{1}{\rho_{H-He}} = \frac{X}{\rho_H} + \frac{Y}{\rho_{He}} + \Delta V(X, Y)$$

$$S_{H-He} = X \cdot S_H + Y \cdot S_{He} + \Delta S(X, Y)$$

$$U_{H-He} = X \cdot U_H + Y \cdot U_{He} + \Delta U(X, Y)$$

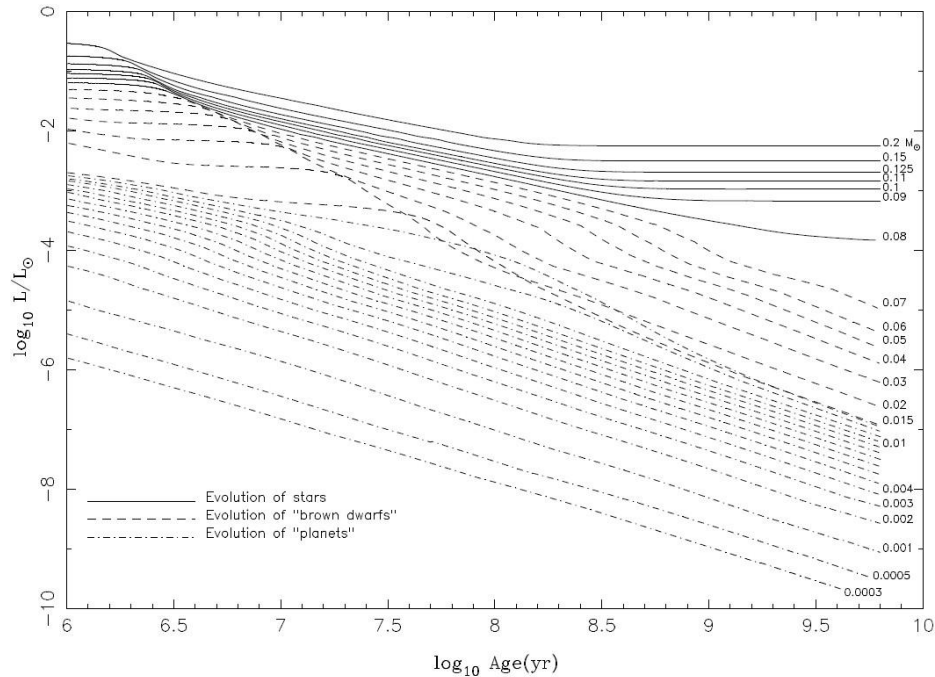
- Non-ideal Mixing terms provided by **Howard & Guillot (2023)**
- We expect: **CMS19 + HG23 = CD21**



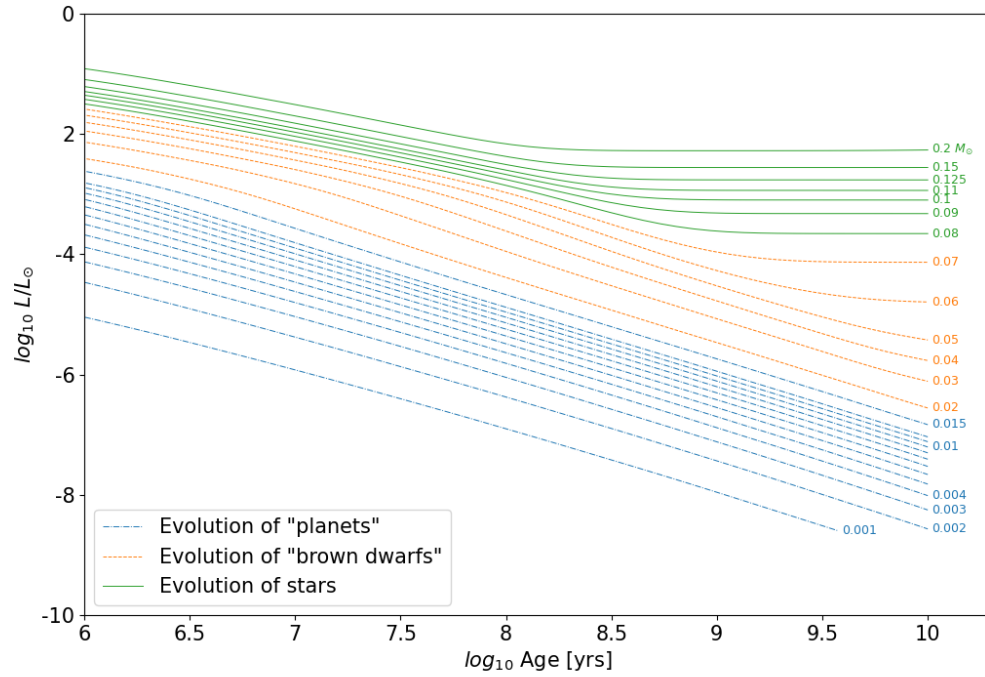
Models use  $X=0.725$ ,  $Y=0.275$  and a mass of  $0.006 M_{\text{sun}}$ . Profiles taken at 5 Gyr

# Modelling Stars and Planets With CEsam2k20

## ii. Benchmarking Against Other Models



*Evolutionary tracks of luminosity versus age from Burrows et al. (1997).*



*Evolutionary tracks of luminosity versus age from this work.*

- **Stars:** Plateau  $\rightarrow$  Hayashi drop  $\rightarrow$  H-burning (main sequence)
- **Brown dwarfs:** Sporadic D-burning slows early cooling
- **Planets:** Steady cooling, no fusion phase

# Conclusion

## i. Completed Work

### Cesam2k20 now Includes:

- ✓ H/He Equations of State Adapted to Planetary Modelling
- ✓ Opacity tables Suited for Planets → *Siebenaler & Miguel (2026)*
- ✓ Planetary Atmospheres With Stellar Irradiation Based on the *Guillot (2010)* Model
- ✓ The Option to Include a Simple Isothermal Planetary Core

## ii. Upcoming Work

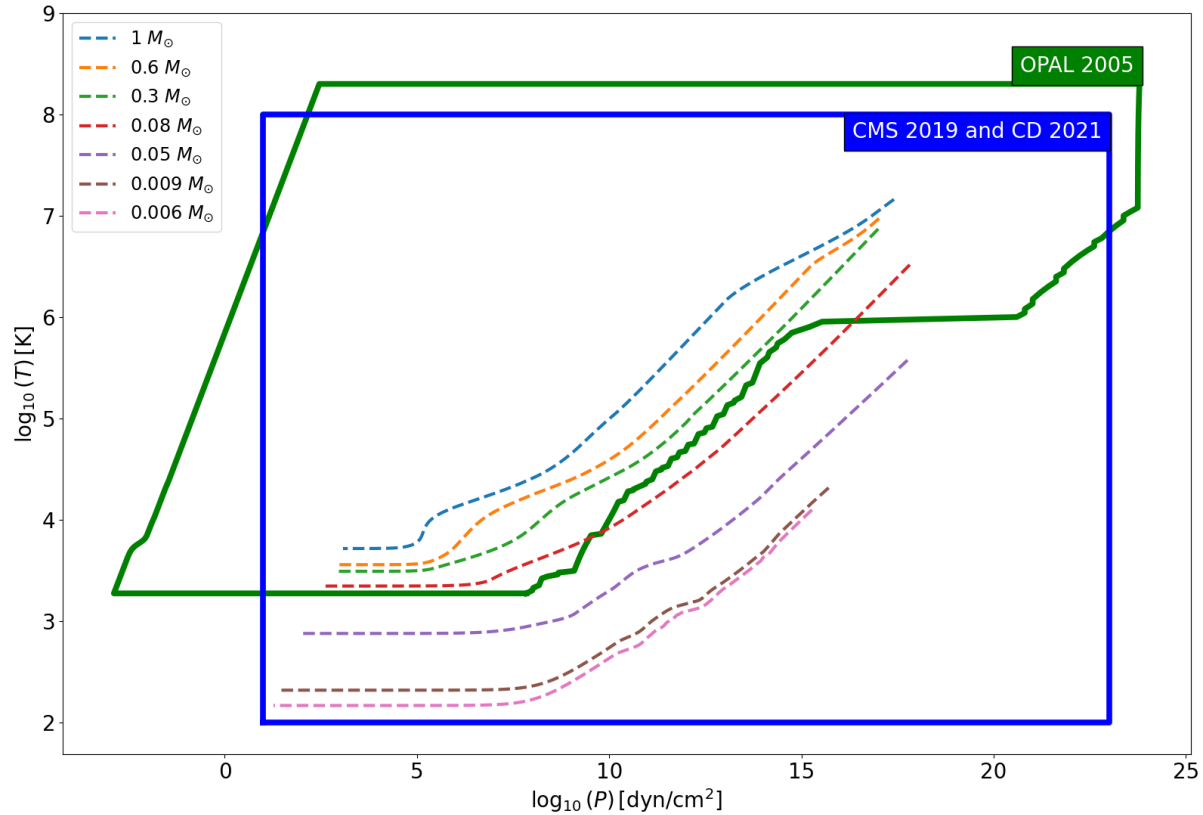
### Part of PLATO WP116100 – Composition & Formation of Gas & Ice Giants

- Extend EoS Capabilities to Include Heavy Elements
- Continue Exploring a New EoS Table Valid for Both Stars and Planets
- Add Deuterium Burning for Brown Dwarfs
- Benchmark Against Real Exoplanets, Ideally in the PLATO Field of View

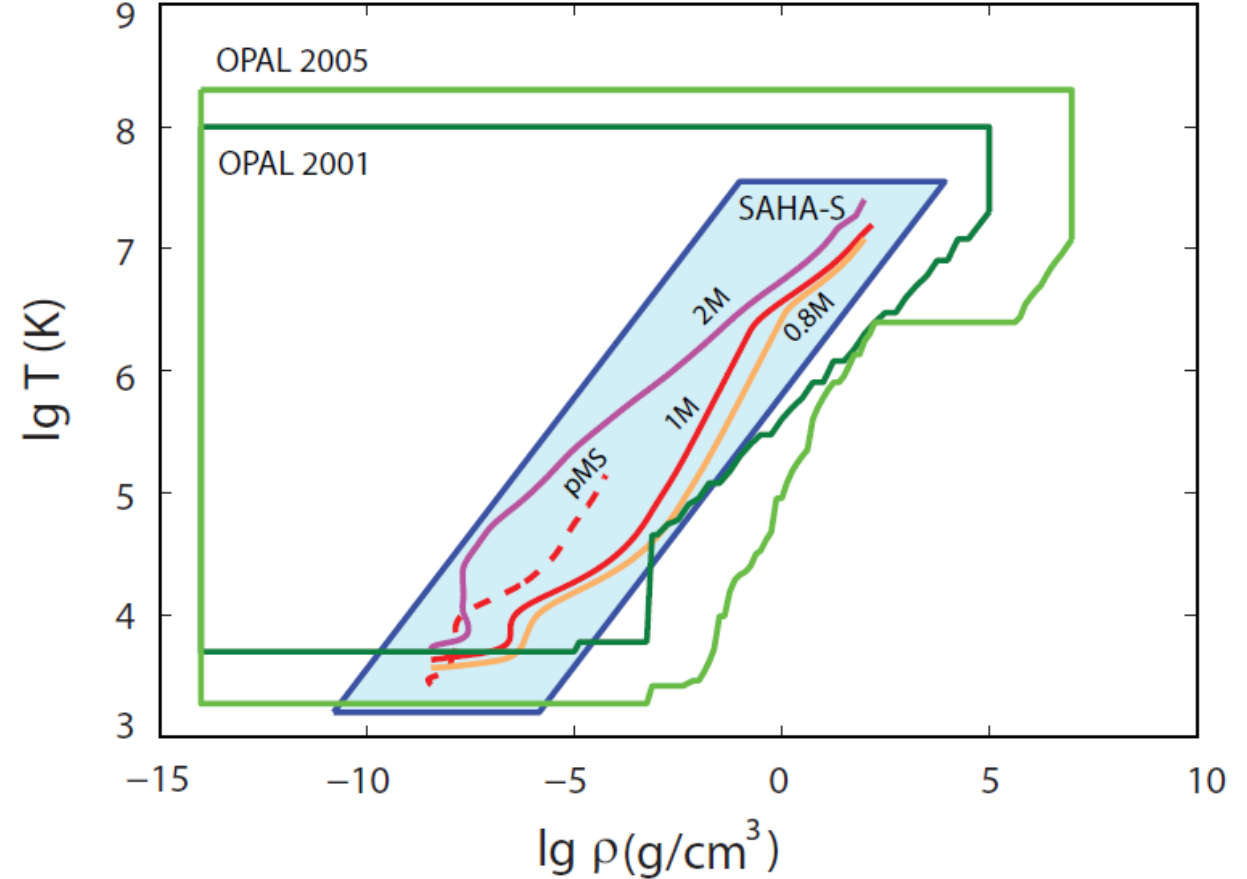
# APPENDICES

---

# Appendix A: EoS Routines of CESAM2k20



Validity domains of CMS 2019, CD 2021 and OPAL 2005 EoS tables in the  $\log P - \log T$  plane, with mass tracks from 1 to  $0.006 M_{\odot}$



Validity domains of EoS tables already present in CESAM2k20, with mass tracks from 2 to  $0.8 M_{\odot}$  (taken from Baturin et al. 2017)

# Appendix B: Numerical Implementation of New EoS Tables

## Columns of the CMS19 and CD21 EoS tables:

log T [K]   log P [GPa]   log rho [g/cc]   log U [MJ/kg]   log S [MJ/kg/K]   dlrho/dlT\_P   dlrho/dlP\_T   dls/dlT\_P   dls/dlP\_T   grad\_ad

**Tables provided:** Pure Hydrogen, Pure Helium, Pre-mixed H/He mixtures

## Variables computed by the CESAM2k20 EoS routines:

$\rho$	$u$	$\delta$	$c_P$	$\nabla_{\text{ad}}$	$\alpha$	$\beta$
Density	Internal energy	Thermal expansivity	Specific heat at constant P	Adiabatic gradient	Isothermal compressibility	Radiation gas pressure ratio
		$\Gamma_1$		$S$		
		First adiabatic index		Entropy (not always computed)		

### • Computed directly from EoS tables:

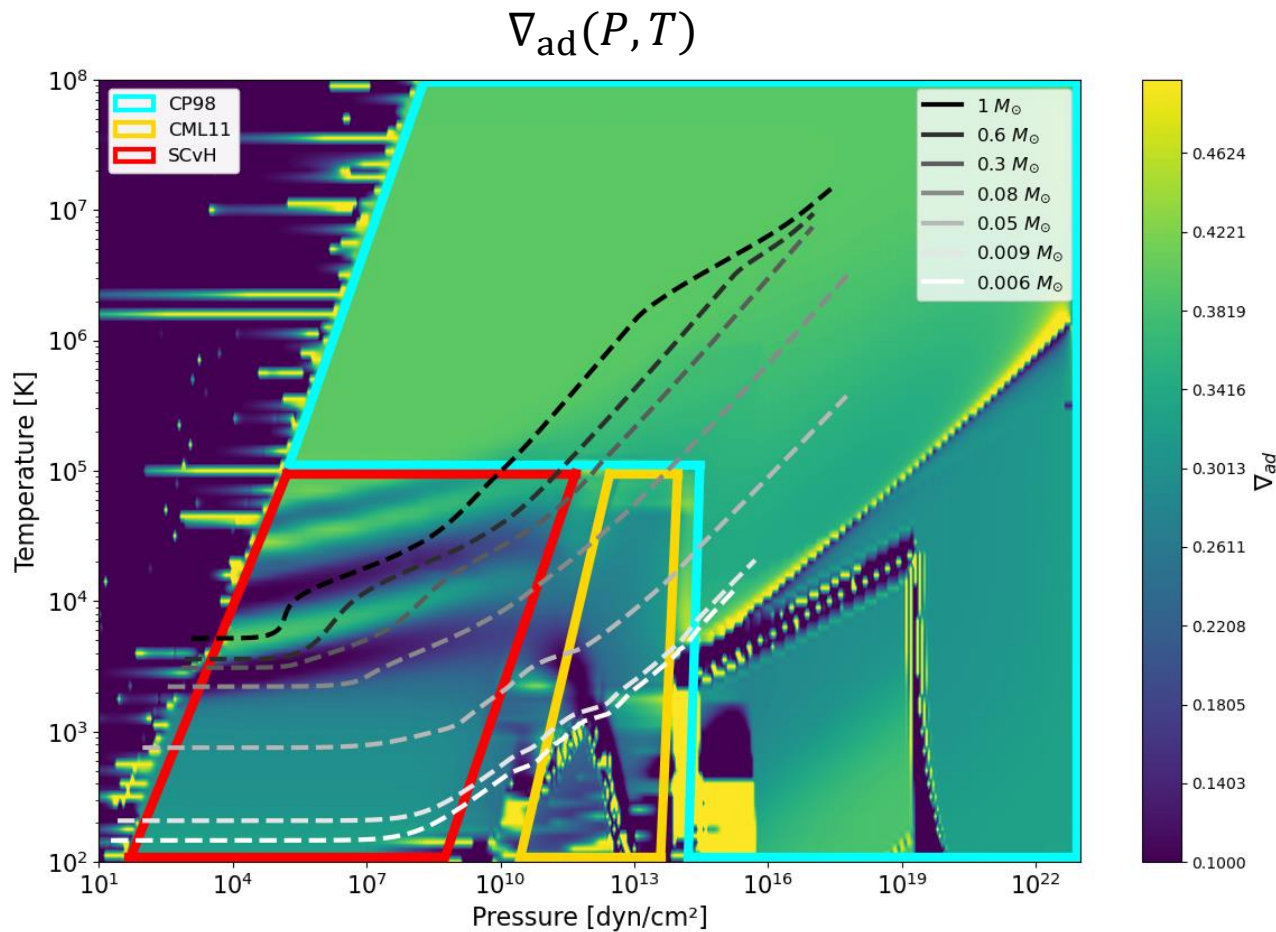
Density  
Internal energy  
Thermal expansivity  $\rightarrow \delta = - \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_P$   
Entropy  
Adiabatic gradient  
Isothermal compressibility  $\rightarrow \alpha = \left( \frac{\partial \ln \rho}{\partial \ln P} \right)_T$

### • Computed with thermodynamic relations:

Specific heat at constant P:  $c_P = T \left( \frac{\partial S}{\partial T} \right)_P$   
Radiation gas pressure ratio:  $\beta = 1 - \frac{a T^4}{3 P}$   
First adiabatic index:  $\Gamma_1 = \left( \frac{\partial \ln P}{\partial \ln \rho} \right)_S = \frac{1}{\alpha - \delta \nabla_{\text{ad}}}$

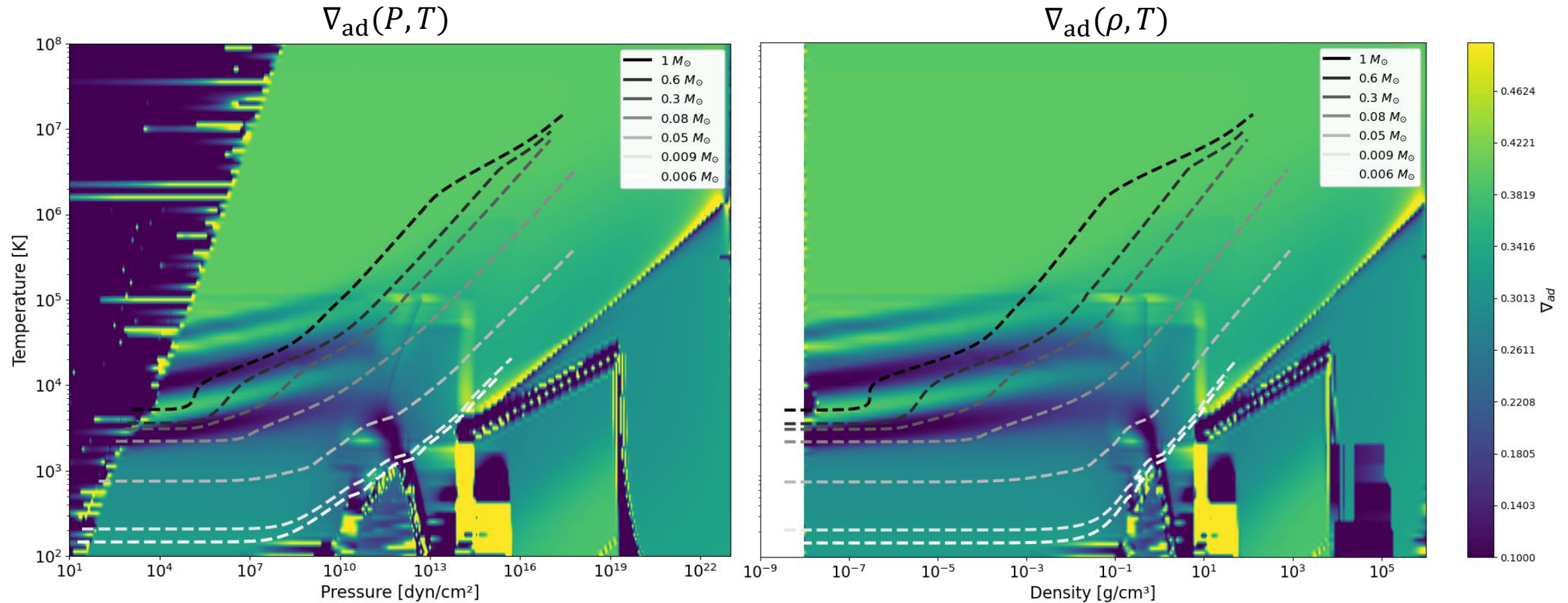
# Appendix C: Contents of CMS19 and CD21

Taken from Chabrier  
et al. (2019)

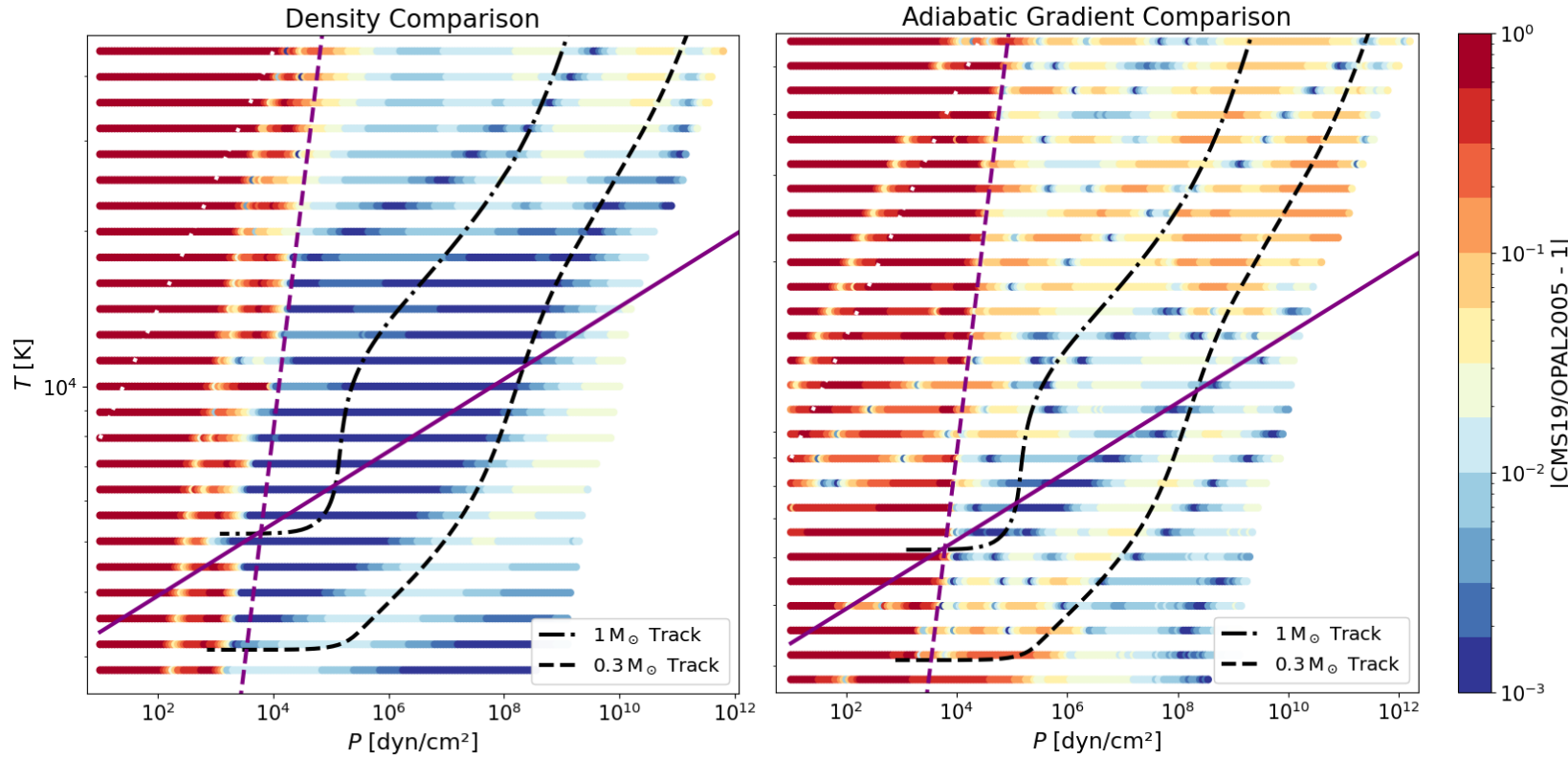


- **CP98** : Chabrier & Potekhin (1998)
  - ✓ Physical Picture → based on fundamental particles
  - ✓ Fully ionized electron-ion plasmas
  - ✓ Uses the One-Component Plasma (OCP) model
  - ✓ Becomes less effective in the partially ionized regime
- **SCvH** : Saumon, Chabrier & van Horn (1995)
  - ✓ Chemical Picture → based on pre-formed chemical species
  - ✓ Includes partial ionization and molecular dissociation
  - ✓ Tends to underestimate pressure dissociation at high densities
- **CML11**: Caillabet, Mazevet & Loubeyre (2011)
  - ✓ Physical Picture → based on fundamental particles
  - ✓ Combination of different ab initio methods such as QMD (Holst et al. (2008)), CEIMC (Morales et al. (2010)), PIMC (Militzer & Ceperley (2000)), and supplemented by additional QMD calculations from Chabrier et al. (2019).

# Appendix D: Extrapolation of the P-T EoS Tables



# Appendix D: Extrapolation of the P-T EoS Tables



extrapolation process for CMS19. The dotted purple line is the pressure threshold line and the full purple line is the extrapolation line. The value of  $P_0$  was chosen arbitrarily for illustration.

Extrapolation line equation:

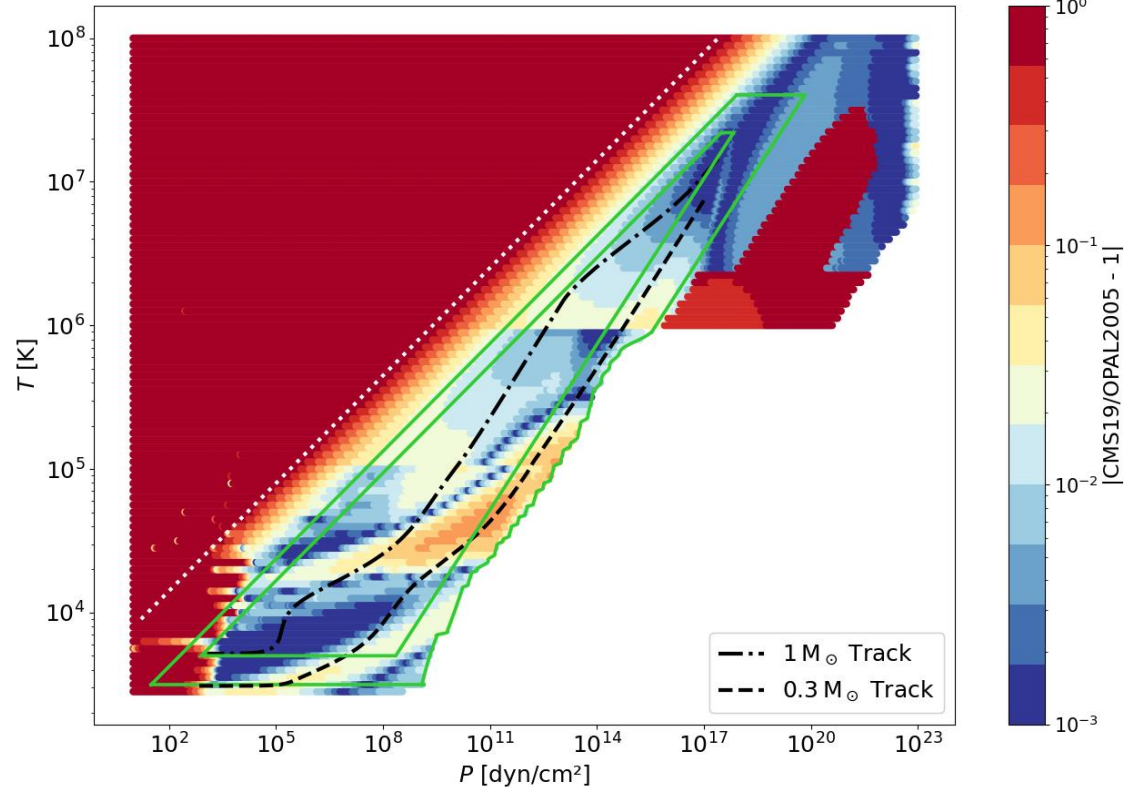
$$T_0 = T \left( \frac{P}{P_0} \right)^{-0.07}$$



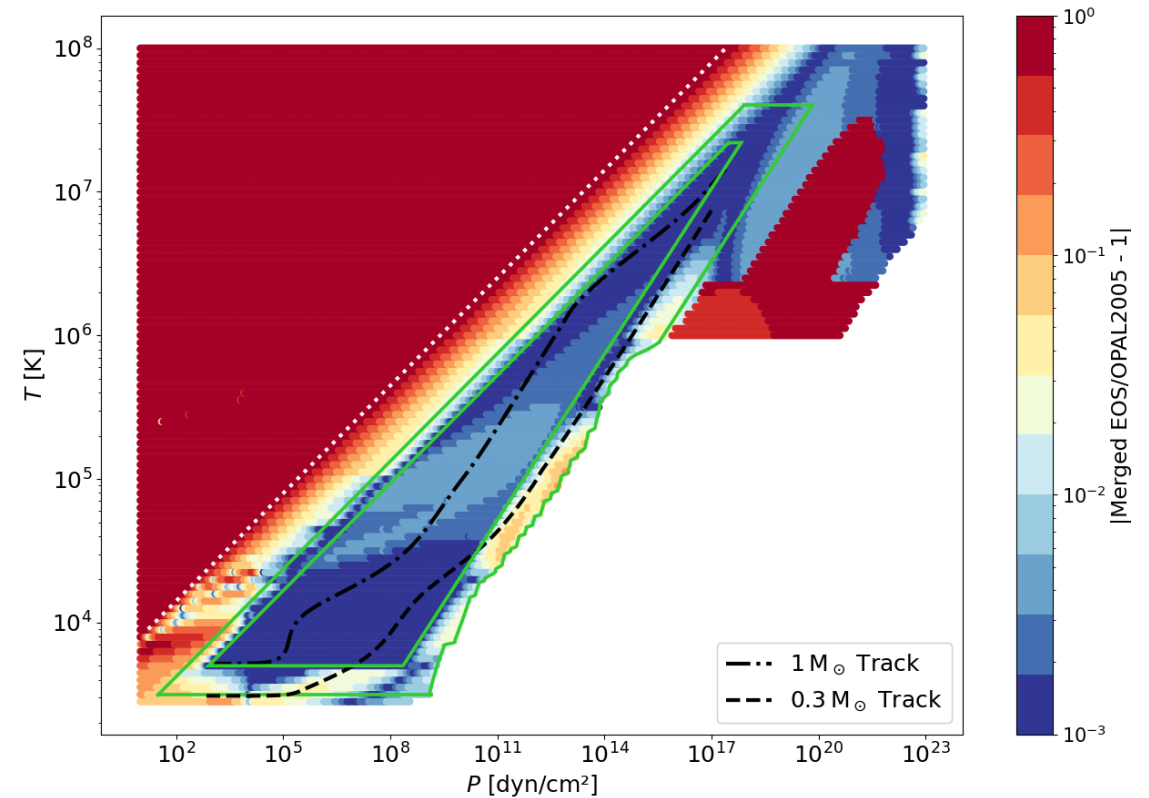
$$\begin{cases} Y(P, T) \rightarrow Y(P_0, T_0) \\ \frac{\partial Y}{\partial P} \rightarrow -0.07 \frac{T_0}{P} \left( \frac{\partial Y}{\partial T_0} \right)_{P_0} & \frac{\partial Y}{\partial T} \rightarrow \left( \frac{\partial Y}{\partial T_0} \right)_{P_0} \left( \frac{P}{P_0} \right)^{-0.07} \end{cases}$$

# Appendix E: Merging OPAL2005 and CMS19

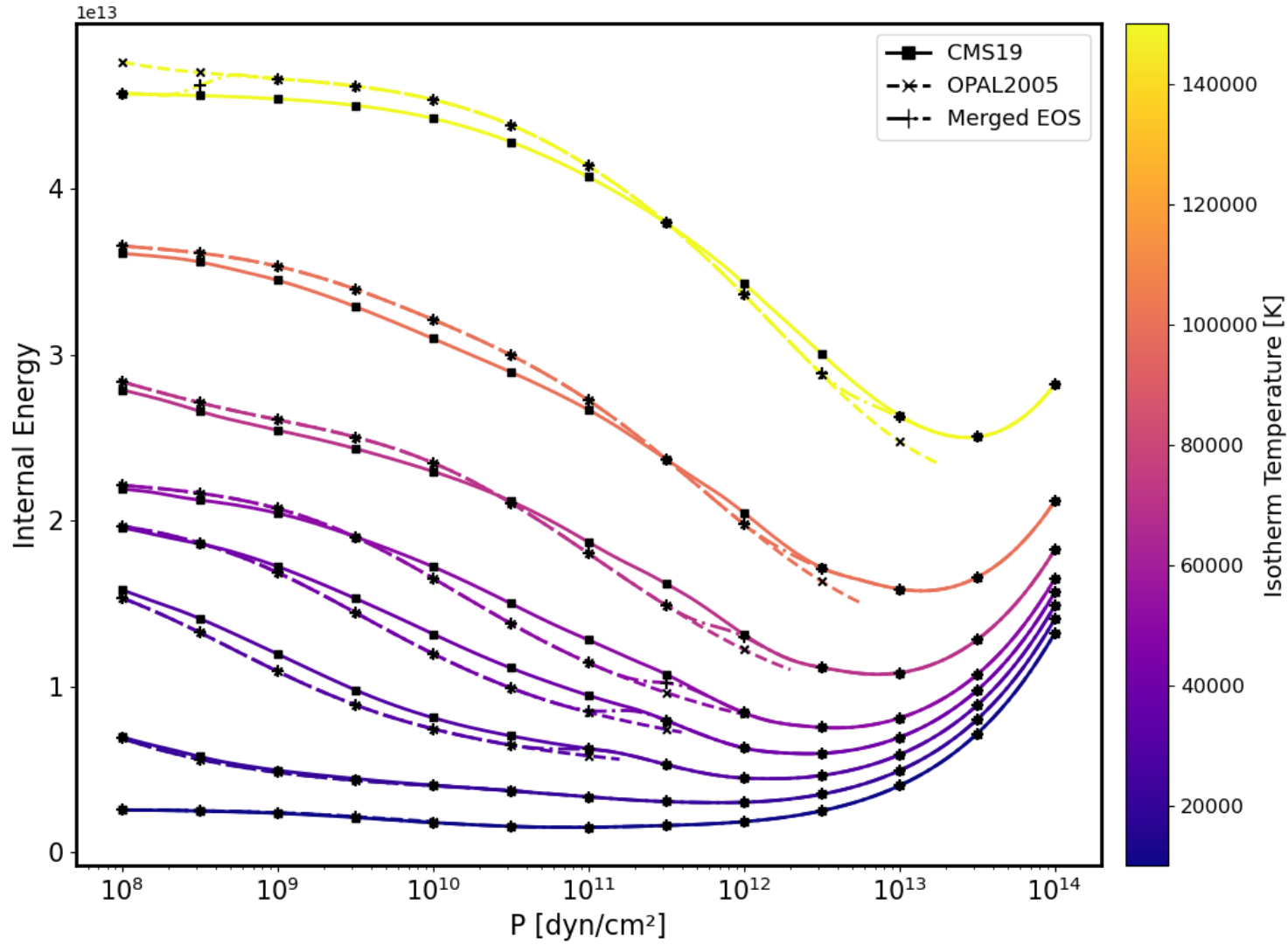
$u(P, T)$ , default



$u(P, T)$ , merged



# Appendix E: Merging OPAL2005 and CMS19



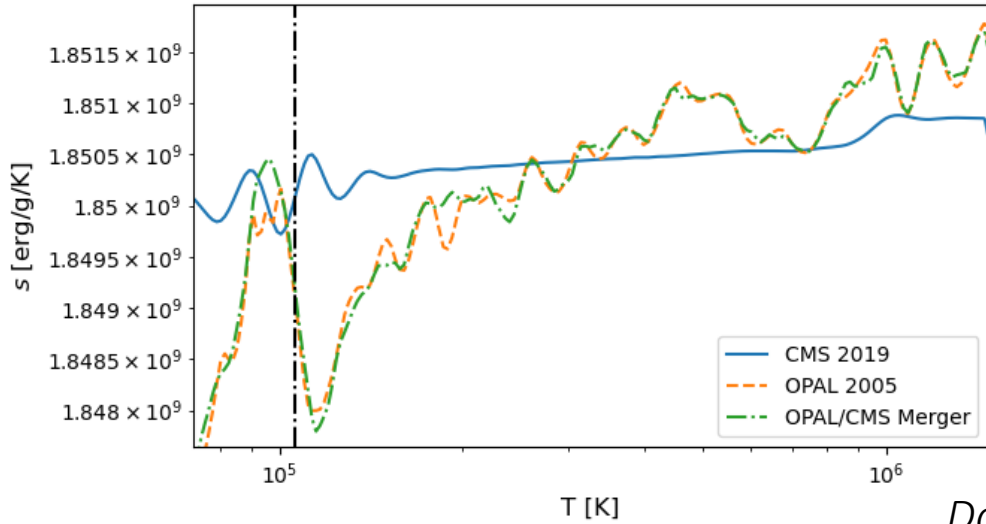
## Strengths:

- Merging is quite smooth
- No visible oscillations during the mixing phase

## Weaknesses:

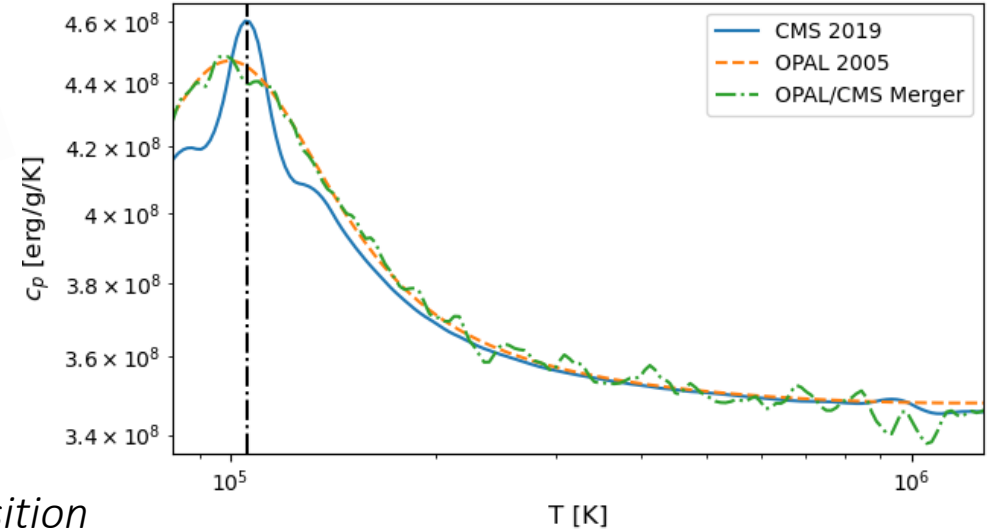
- Non-physical behaviour caused by the bounds of the merging?
- More effective weight functions available?

# Appendix E: Merging OPAL2005 and CMS19



$$c_P = T \left( \frac{\partial s}{\partial T} \right)_P$$

*Dash-dotted line: SCvH Transition*



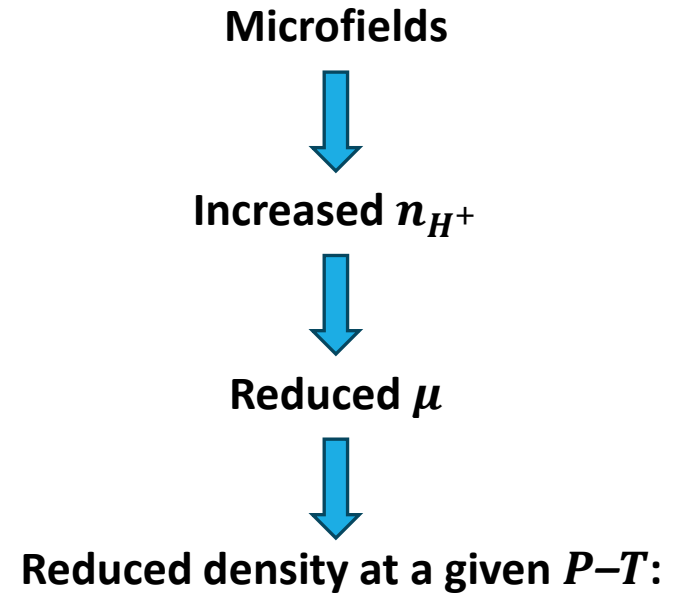
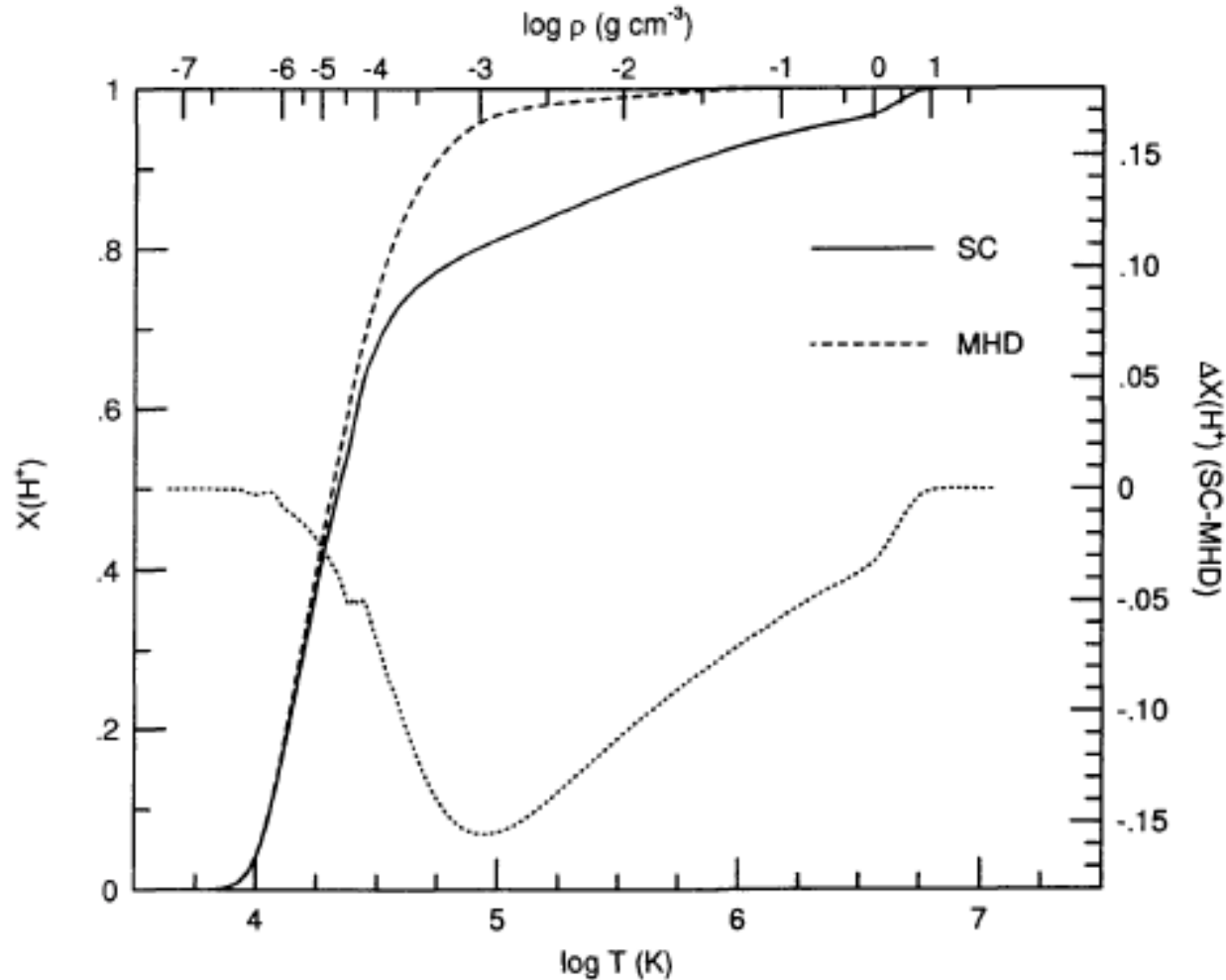
Relative amplitude of oscillations  $\approx 4 \times 10^{-4}$

Relative amplitude of oscillations  $\approx 2 \times 10^{-2}$

## OPAL2005 Entropy and density oscillate when using a PT track from a CESAM2k20 Model

- Problems during the rho-T to P-T switch process?

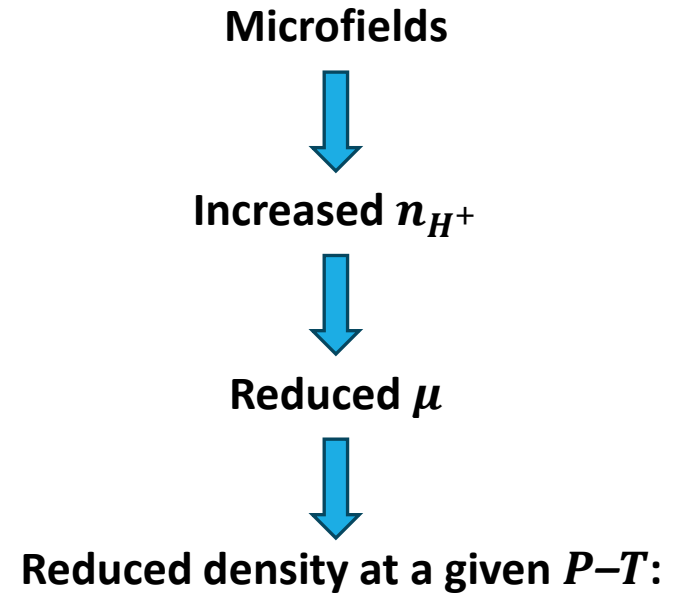
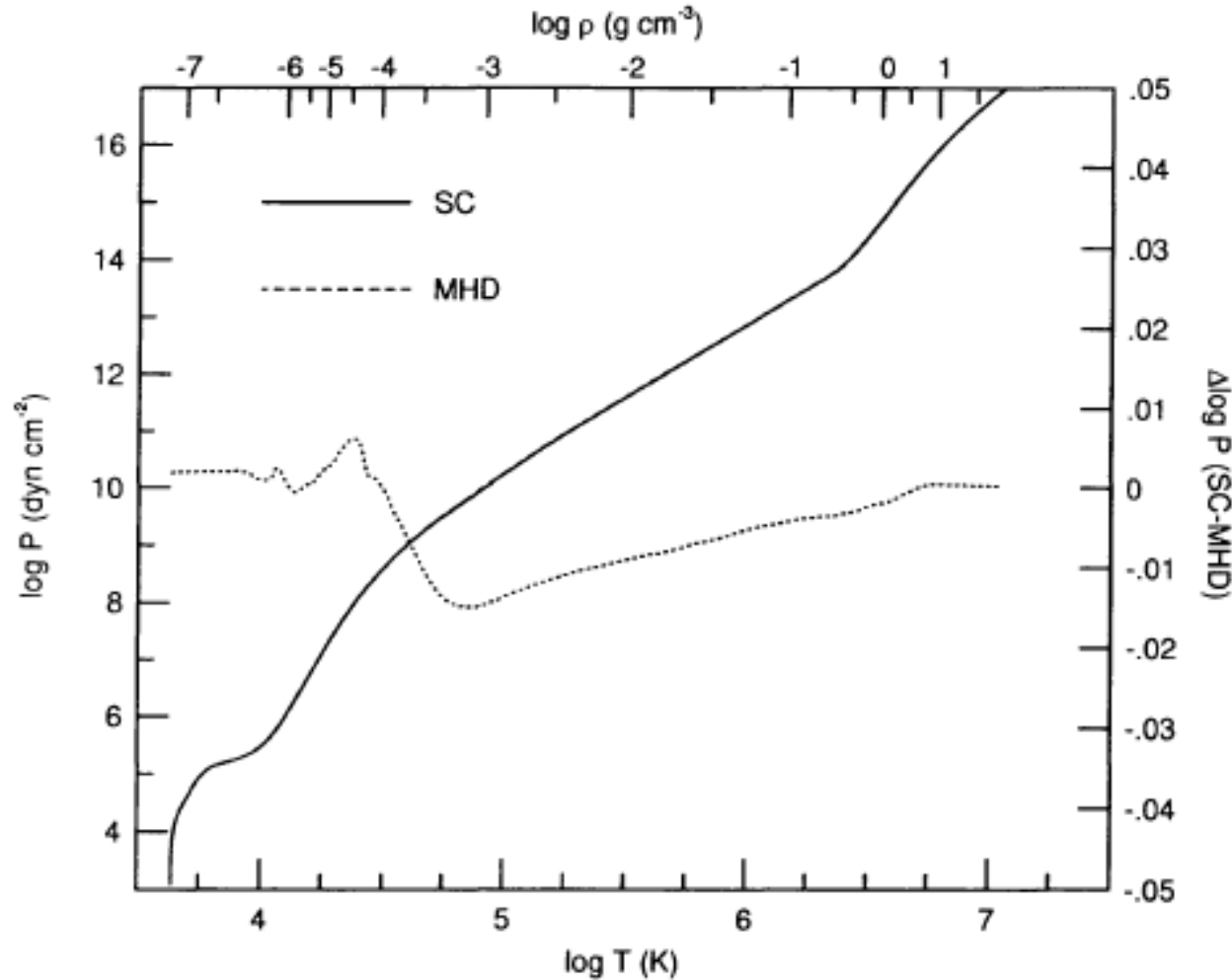
# Appendix F: Microfield effects and their consequences



$$\rho(P, T) = \frac{\mu m_u}{k_B} \frac{P_{\text{gas}}}{T}$$

Visualization of microfield effects along a solar  $P-T$  profile: MHD includes microfield effects. Comparisons shown for  $H^+$  mass fraction (a), pressure (b) and the first adiabatic index (c). Figures taken from Saumon et al. (1995).

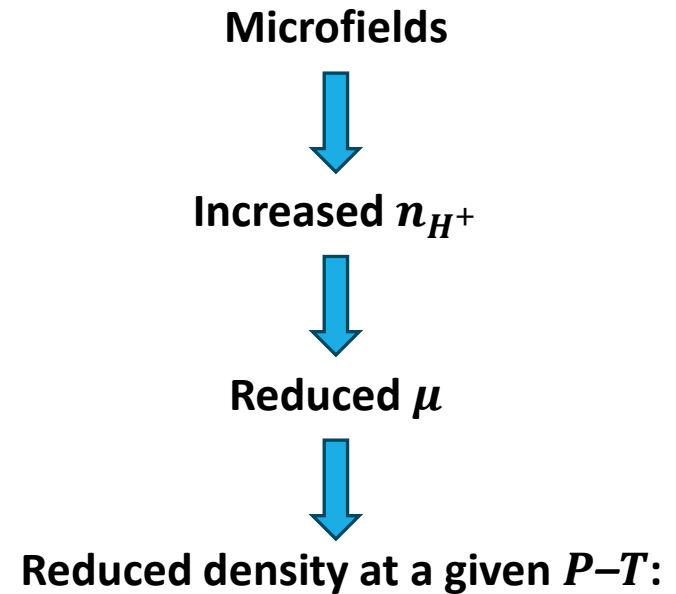
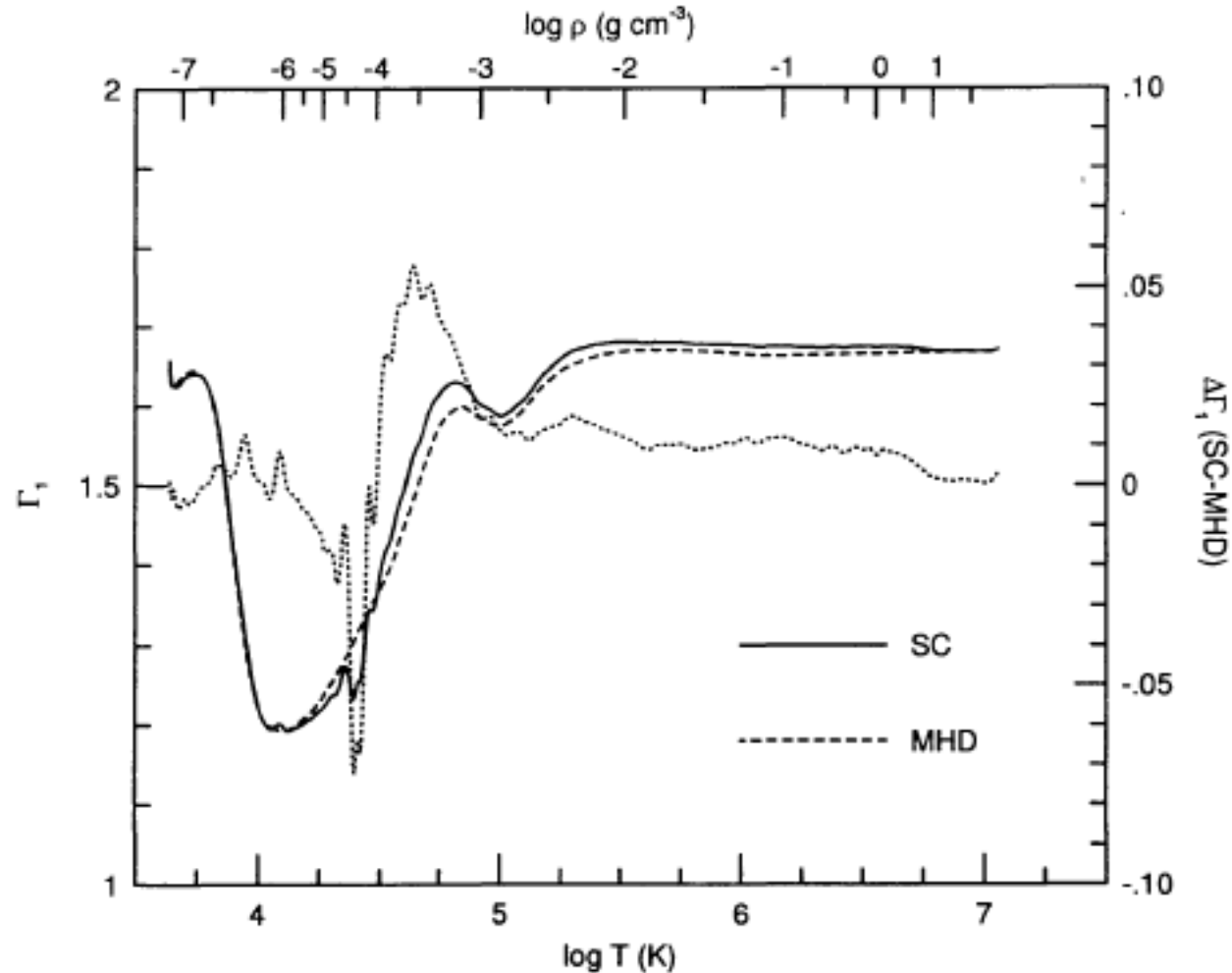
# Appendix F: Microfield effects and their consequences



$$\rho(P, T) = \frac{\mu m_u}{k_B} \frac{P_{\text{gas}}}{T}$$

Visualization of microfield effects along a solar  $P-T$  profile: MHD includes microfield effects. Comparisons shown for  $H^+$  mass fraction (a), **pressure (b)** and the first adiabatic index (c). Figures taken from Saumon et al. (1995).

# Appendix F: Microfield effects and their consequences



$$\rho(P, T) = \frac{\mu m_u}{k_B} \frac{P_{\text{gas}}}{T}$$

Visualization of microfield effects along a solar  $P-T$  profile: MHD includes microfield effects. Comparisons shown for  $H^+$  mass fraction (a), pressure (b) and the *first adiabatic index* (c). Figures taken from Saumon et al. (1995).

# Appendix G: Adiabatic Gradient Differences

