Multiphase Equations of State for Exoplanet Interiors

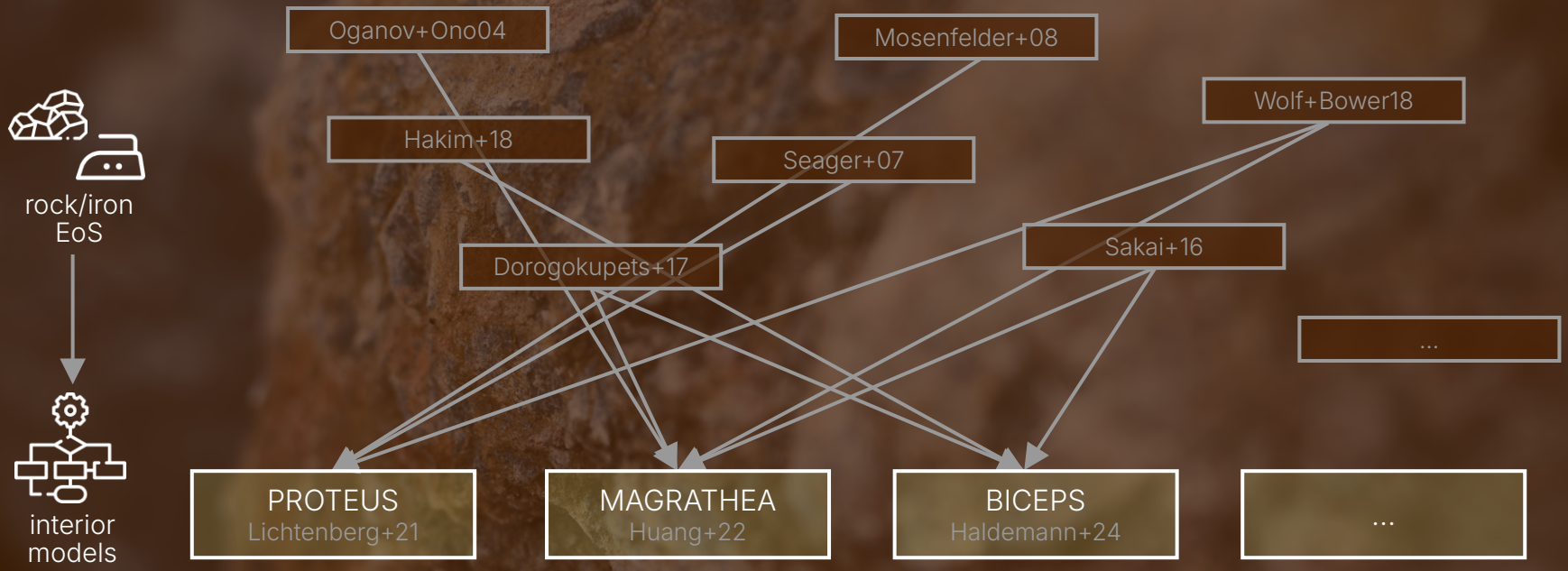
Mara Attia (they/them), SNSF Postdoc.Mobility Fellow
with Tim Lichtenberg, Ema Jungová, Mariana Sastre

Kapteyn Astronomical Institute (Groningen, NL)

Swiss National Science Foundation

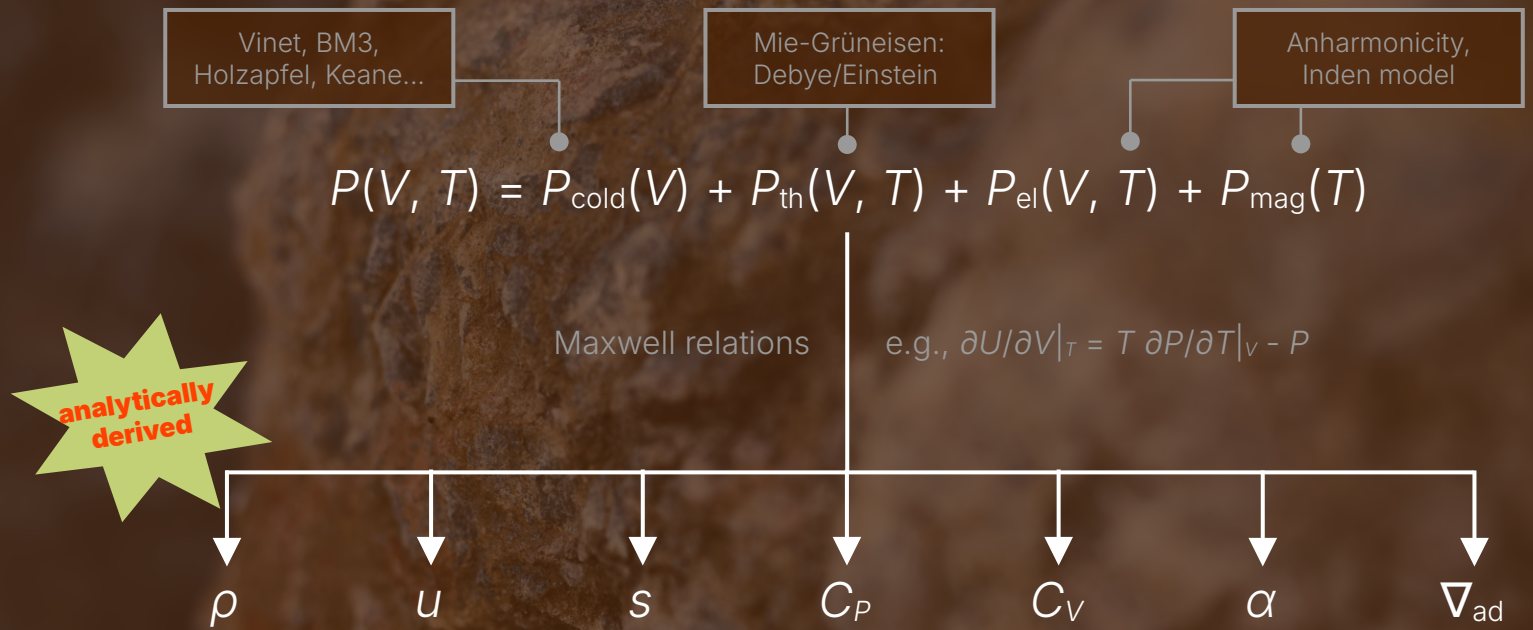
university of groningen

The EoS Implementation Problem



Source: https://github.com/maraattia/PALEOS
EoS tables: https://zenodo.org/records/19000316
Mass-radius tables: https://zenodo.org/records/19221215

One Function, Seven Quantities



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Three Materials, 17 Phases

Fe
(planet core)

solid, low pressure
- α-bcc
- δ-bcc
- γ-fcc

solid, high pressure
- ϵ-hcp

liquid

$MgSiO_3$
(planet mantle)

solid, low pressure
- lp/hp clinoenstatite
- orthoenstatite
- bridgmanite

solid, high pressure
- postperovskite

liquid

H_2O
(volatiles/ice)

10+ phases via AQUA
(Haldemann+20)

corrected entropy

one-stop shop →

zenodo
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- no installation
- no dependency
- just interpolate

PALEOS

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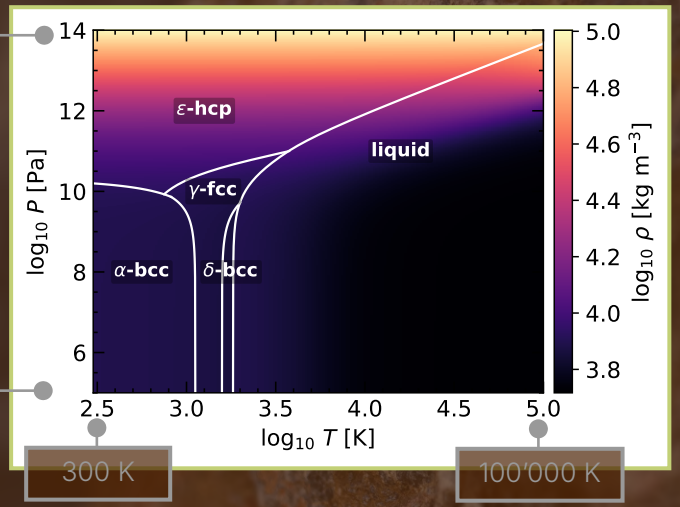
Full Phase Coverage

100 TPa

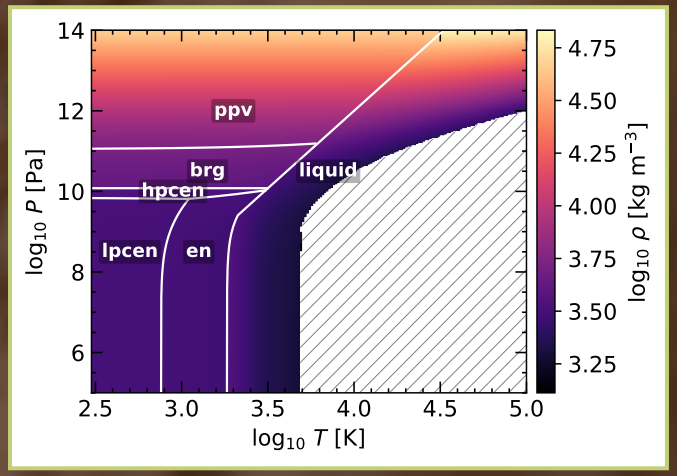
Fe

1 bar



300 K

100'000 K



$MgSiO_3$

Thermodynamic consistency:

$$\Delta \equiv 1 - \rho^2 \left.\frac{\partial S}{\partial P}\right|_T \Big/ \left.\frac{\partial \rho}{\partial T}\right|_P$$

$|\Delta| < 1\%$ → consistent (Haldemann+20)

Fe: 98% below 1% (median = 1.4×10^{-7})

$MgSiO_3$: 100% below 1% (median = 3.5×10^{-8})

PALEOS

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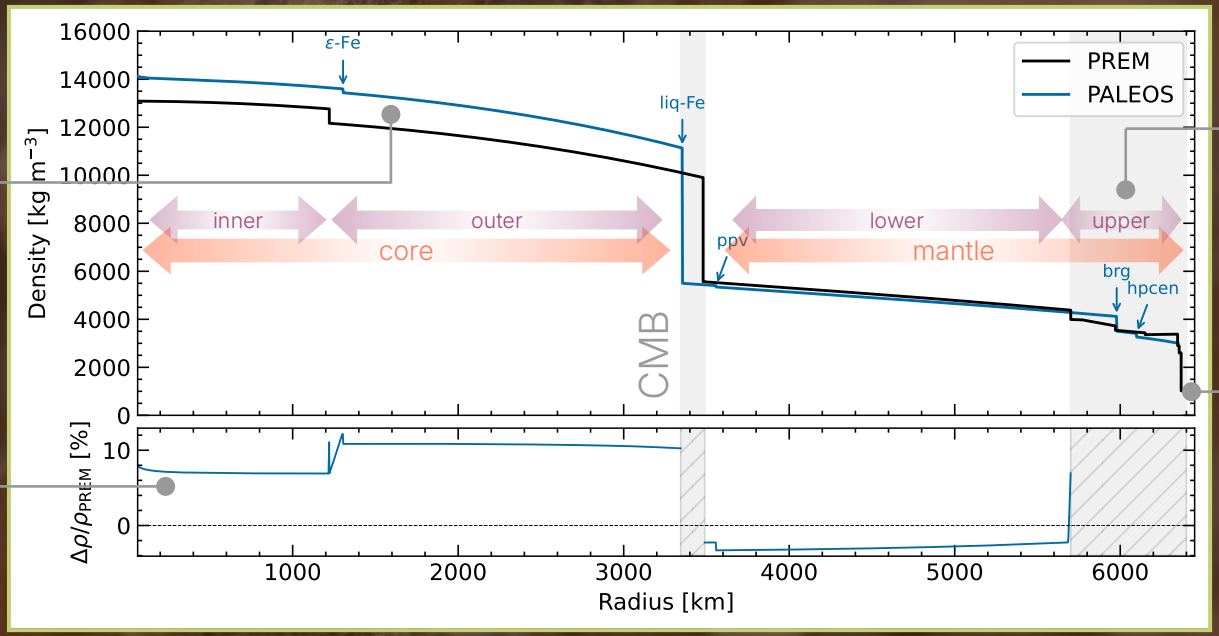
Does it Work? Ask the Earth.

solid inner core
+ liquid outer core

8 – 10% overshoot
(no light elements)



pyroxenes: proxies for
olivine-dominated
upper mantles

radius recovered
to ~ 0.33%

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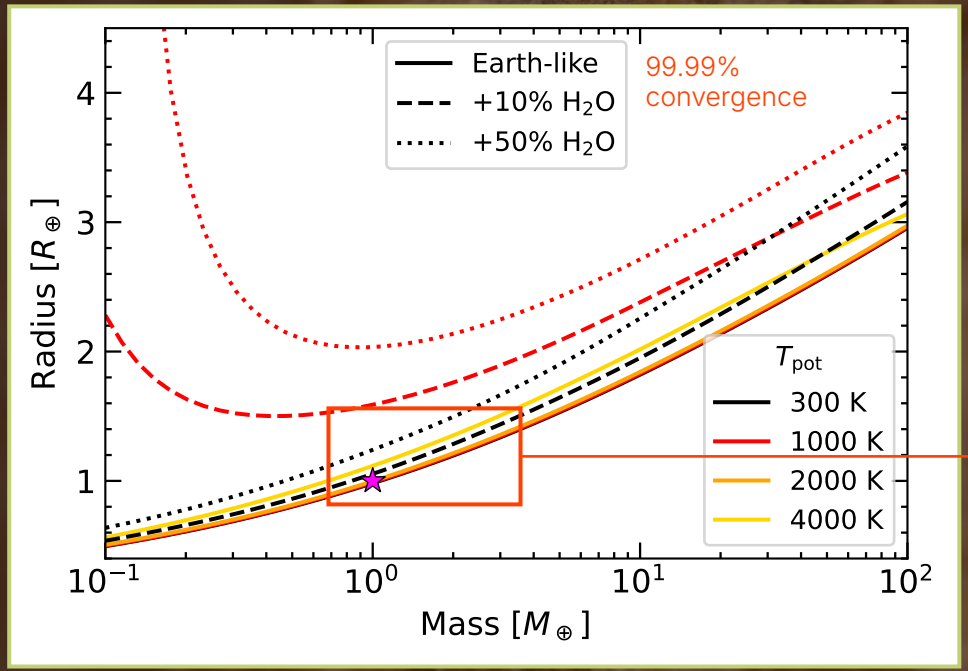
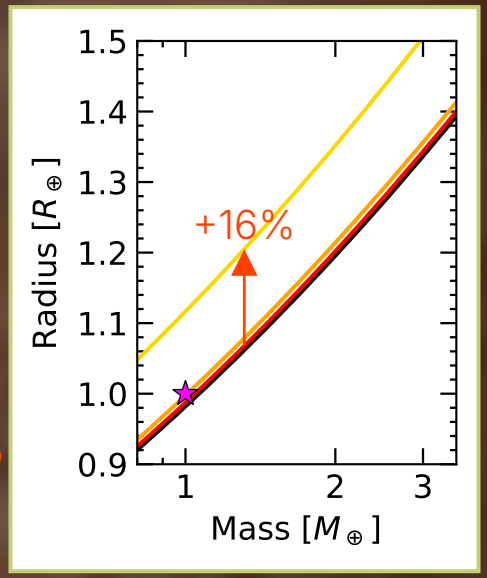
17,900 Models at Finite Temperature





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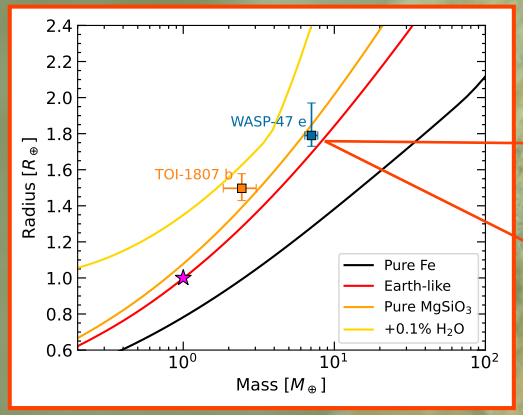
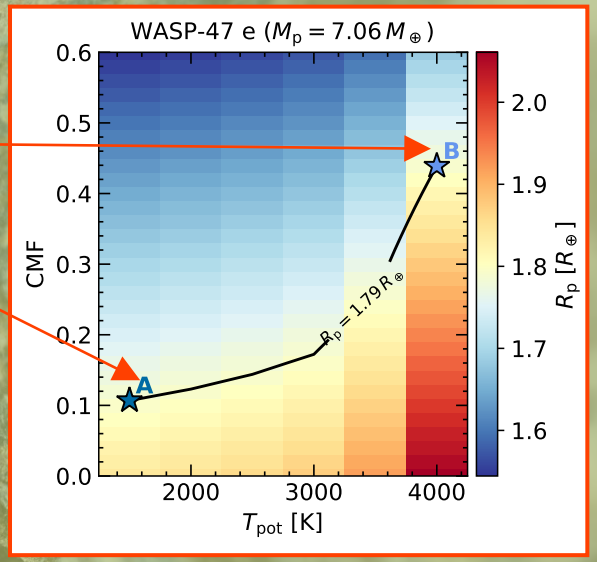
Same Planet, Two Worlds





Solution A
(cold)

CMF = 0.107

T_{pot} = 1500 K

Solid mantle

Solid core

→ geologically dead/frozen

Solution B
(hot)

CMF = 0.439

T_{pot} = 4000 K

Magma ocean

Outer liquid + inner solid core

→ geologically active, dynamo

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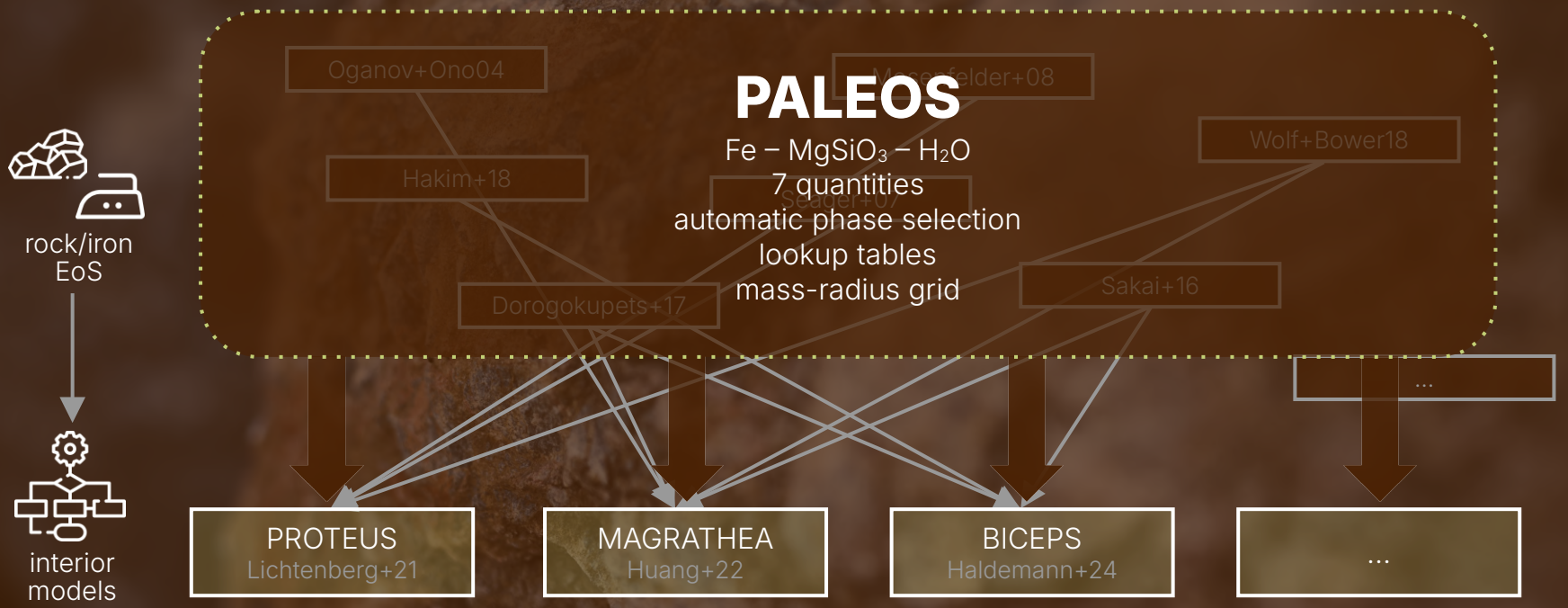
Where PALEOS Fits



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What's Missing (Yet)

Fe alloys
(FeS, Fe-Si)

Oxides
(SiO_2, MgO)

Volatiles
(CO_2, NH_3)

rock/iron
EoS

interior
models

Oganov+Ono04

Hakim+18

Dorogokupets+17

Wolf+Bower18

Sakai+16

...

PALEOS

Fe – $MgSiO_3$ – H_2O
7 quantities
automatic phase selection
lookup tables
mass-radius grid

PROTEUS
Lichtenberg+21

MAGRATHEA
Huang+22

BICEPS
Haldemann+24

...

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Use PALEOS

1. Download the EoS tables.

2. Integrate them to your model.

```python
# Interpolate
interp = RegularGridInterpolator((log_P, log_T), rho)
rho_query = interp([[np.log10(100e9), np.log10(4000)]])[0]
```

3. Extend them.

your P(V, T) + our framework = next PALEOS table

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